

# OPTICS AND THERMAL PHYSICS

## UNIT I

Refraction through a thin lens, power of a lens, effective focal length of two thin lenses in and out of contact, chromatic and spherical aberration and their removal, refraction of light through a thin prism, dispersion of light and dispersive power of a prism.

## UNIT II

Conditions for interference, Young's experiment – theory, Newton's rings – determination of wavelength of light (theory & expt), air wedge, determination of diameter for thin wire (theory & expt)

## UNIT III

Fresnel and Fraunhofer diffraction, theory of plane transmission grating- wavelength determination, dispersive and resolving power of a grating – Zone plate- theory

## UNIT IV

Double refraction, Nicol prism, quarter wave and half wave plate, production, detection and analysis of plane circularly and elliptically polarized light- specific rotation – Laurent's half shade polarimeter

## UNIT V

Kinetic theory of gases: concept of heat and temperature – ideal and perfect gas- Kinetic theory of gases- Expression for a pressure of a gas – interpretation of temperature - Gas laws – Gas equation – Avogadro's hypothesis- Transport of momentum – Transport of energy- Transport of matter – behavior of gases at high pressure – Vander Waals equation of state – Critical constants – Experimental determination – Porous plug experiment – theory.

### **Books for study**

1. A TEXT BOOK OF OPTICS – N. Subramaniam, Brijilal revised by M.N. Avadhanulu, S. Chand & Co Pvt Ltd, New Delhi
2. OPTICS AND SPECTROSCOPY – R. Murugesan – S. Chand & Co Pvt Ltd, New Delhi
3. HEAT AND THERMODYNAMICS – Brijilal and Subramaniam - S.Chand & Co pvt Ltd, New Delhi

### **Books for Reference**

Physics, Robert Resnick, David Haliday, Jearl Walker Wiley and Sons inc. Sixth edition.

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# UNIT I

## 1.1 Introduction

A lens is a transparent refracting medium bounded by two spherical surfaces. The line joining the centers of curvature of the two surfaces is called the principal axis.

### *First principal focus ( $f_1$ )*

It is that point on the principal axis of the lens, the rays starting from which (convex lens) or appear to converge at which (concave) become parallel to principal axis after refraction from the lens.

### *Second principal focus ( $f_2$ )*

Second principal focus is that point on the principal axis at which the rays parallel to principal axis converge (convex lens) or appear to diverge (concave) after refraction from the lens.

## 1.2 Refraction through a thin lens

Consider a thin convex lens AB of refractive index  $n$  placed in the air medium. The lens has two refracting surfaces. Let  $C_1$  and  $C_2$  are the centers of curvatures of the two refracting surfaces.  $R_1$  and  $R_2$  are the radius of curvature of the surfaces. Both the surfaces are convex towards the rarer medium;  $R_1 = +Ve$   $R_2 = +Ve$ .

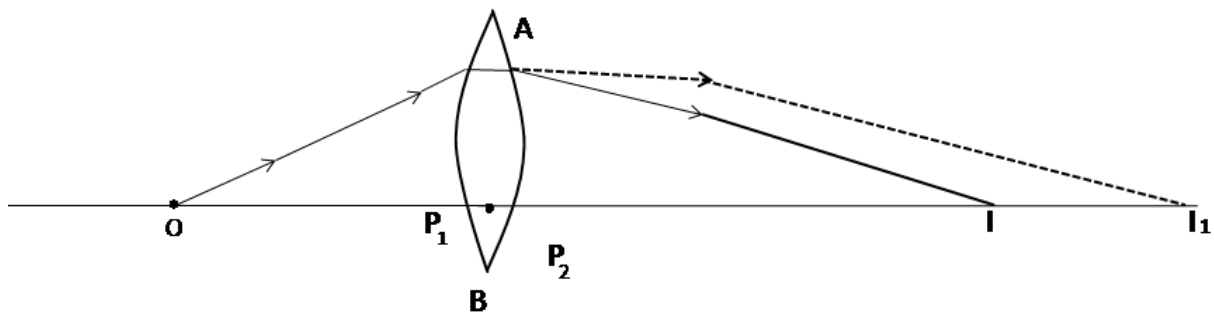


Fig.1.1

Consider a point object O placed on the principal axis. Let OA be the incident ray strikes the first refracting surface at A and refracted along BI<sub>1</sub>. Let the image be formed at I<sub>1</sub>.

Object distance (u) = +u; image distance (v) = +v

By gauss law

$$\frac{n_2}{v} + \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

Where  $v = v_1, n_2 = n; n_1 = 1$  and  $R = R_1$

$$\frac{n}{v_1} + \frac{1}{u} = \frac{n-1}{R_1} \dots\dots\dots (1)$$

Now I<sub>1</sub> acts as the virtual object for the refraction at the second surface and the final image is formed at I.

Object distance I<sub>1</sub>p<sub>2</sub> = -v<sub>1</sub> ; image distance Ip<sub>2</sub> = v

The position of I and I<sub>1</sub> can be interchanged. Hence, the object distance = v and

Image distance = -v<sub>1</sub> and R = R<sub>2</sub>

$$\frac{n}{-v_1} + \frac{1}{v} = \frac{n-1}{R_2} \dots\dots\dots (2)$$

Adding equations (1) and (2)

$$\frac{1}{u} + \frac{1}{v} = (n - 1) \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

$$\frac{1}{f} = (n - 1) \left( \frac{1}{R_1} + \frac{1}{R_2} \right) . \text{ This equation is known as lens maker's formula.}$$

Where  $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ .

### 1.3 Power of a lens

Power of a lens is its ability to converge or diverge the ray of light. The power of a lens is measured by the reciprocal of its focal length. If f is the focal length, power of the lens P =  $\frac{1}{f}$ . The unit of power of lens is dioptre. The power of a convex lens is positive and that for a concave lens is negative.

### 1.4 Effective focal length of two thin lenses in contact

$L_1$  and  $L_2$  are two thin convex lenses of focal length  $f_1$  and  $f_2$  respectively. They are kept in contact with each other so that both have a common principal axis and they act as a single lens of equivalent focal length  $f$ .

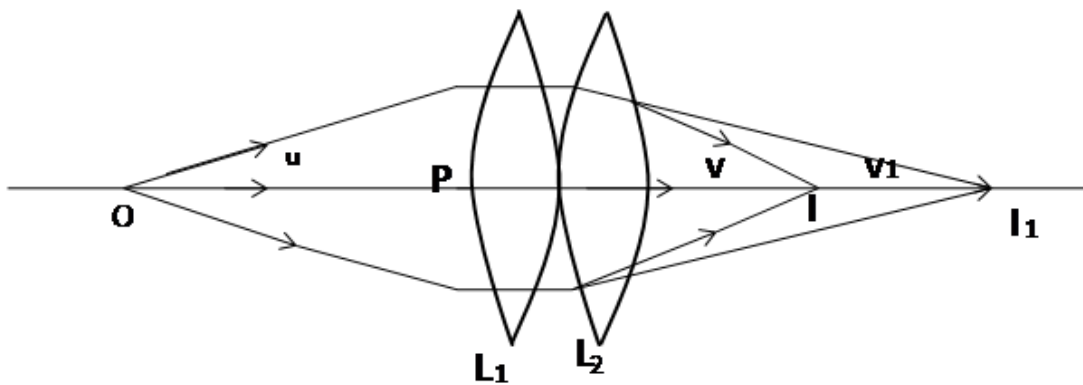


Fig. 1.2

Let  $O$  be an object placed at the distance  $u$  from the lens system on the left side. The image  $I_1$  is formed at a distance  $v_1$  due to the refraction through the lens  $L_1$ .

Object distance ( $u$ ) =  $u$ ; image distance ( $v$ ) =  $v_1$  and  $f = f_1$

The equation  $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$  can be written as

$$\frac{1}{f_1} = \frac{1}{u} + \frac{1}{v_1} \dots\dots\dots (1)$$

Now  $I_1$  acts as a virtual object for the second lens  $L_2$ . The final image  $I$  is formed at a distance  $v$ .

Object distance ( $u$ ) =  $-v_1$  since the object is virtual;

Image distance ( $v$ ) =  $v$  and  $f = f_2$

$$\frac{1}{f_2} = \frac{1}{-v_1} + \frac{1}{v}$$

$$\frac{1}{f_2} = \frac{1}{v} - \frac{1}{v_1} \dots\dots\dots (2)$$

Adding equations (1) and (2)

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

The value of 'f' gives the effective focal length of the combination of lenses.

### 1.5 Effective focal length of two thin lenses out of contact

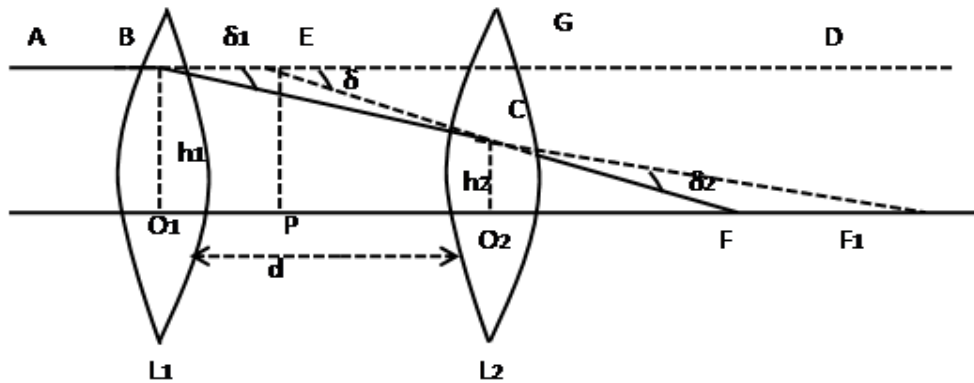


Fig. 1.3

Consider two thin lenses  $L_1$  and  $L_2$  of focal lengths  $f_1$  and  $f_2$  respectively are kept at a distance  $d$  apart. Let the incident ray  $AB$  strikes at  $L_1$  at a height  $h_1$  above the axis. In the absence of  $L_2$ , the ray passes through the principal focus  $F_1$ . The deviation produced by  $L_1$  is given by

$$\delta_1 = \frac{h_1}{f_1} \quad \text{Where } f_1 = O_1F_1 = \text{the focal length of lens } L_1$$

But due to the presence of lens  $L_2$ , this ray incident on  $L_2$  at height  $h_2$  above the axis and the final image is formed at  $F$ . The deviation produced by  $L_2$  is  $\delta_2$ .

$$\delta_2 = \frac{h_2}{f_2} \quad \text{Where } f_2 \text{ is the focal length of the lens } L_2.$$

The incident ray  $AB$  and the emergent ray  $CF$  are produced to meet at  $E$ . The net deviation produced at  $E$  is  $\delta$ . Then, a single convex lens is placed in the position  $PE$  will provide the same effect on a parallel beam as it is produced by the two lenses jointly. i.e.) the final image is at  $F_2$ . The distance  $PF$  is the equivalent focal length  $f$  of the lens system. The deviation produced by the equivalent lens is given by

$$\delta = \frac{PE}{f} = \frac{h_1}{f}$$

$$\delta = \delta_1 + \delta_2$$

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_2}{f_2} \dots\dots\dots(1)$$

From figure

$$h_2 = GO_2 - GC \qquad GC = \delta_1 d; \quad GO_2 = h_1$$

$$h_2 = h_1 - \delta_1 d \qquad \delta_1 = \frac{h_1}{f_1}$$

$$h_2 = h_1 - \frac{h_1}{f_1} d$$

$$h_2 = h_1 \left(1 - \frac{d}{f_1}\right) \dots\dots\dots(2)$$

Substituting eqn (2) in eqn (1), we get

$$\frac{h_1}{f} = \frac{h_1}{f_1} + \frac{h_1 \left(1 - \frac{d}{f_1}\right)}{f_2}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \left(1 - \frac{d}{f_1}\right)$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\frac{1}{f} = \frac{f_1 + f_2 - d}{f_1 f_2}$$

This gives the equivalent focal length of the combination of the lenses.

$$f = \frac{f_1 f_2}{\Delta} \quad \text{Where } \Delta = f_1 + f_2 - d \text{ is known as the optical interval between the two lenses.}$$

### 1.6 Aberration in a lens

The deviation in the size, shape, position and color in the actual image produced by a lens in comparison of the object are called aberration produced by a lens.



### 1.6.1 Spherical aberration in a lens

This aberration is due to large aperture of the lenses. The peripheral and the central rays from the object do not meet after refraction at the same point. The paraxial and marginal rays form the image at different places. Hence the image is not sharp at any point on the axis. This defect is known as the spherical aberration.

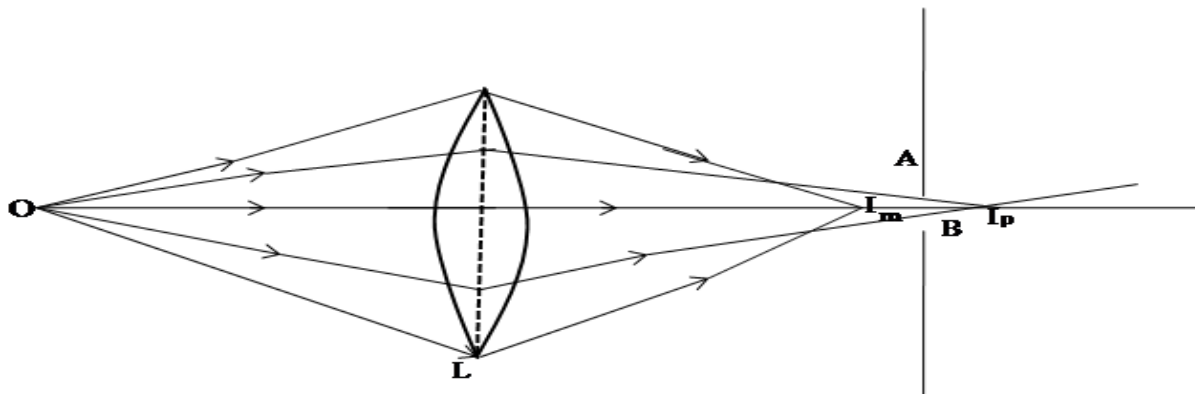


Fig. 1.4

Consider the refraction of light through a lens. Let O is the point object.  $I_p$  and  $I_m$  are the images formed by the paraxial and marginal rays respectively. The distance between  $I_p$  and  $I_m$  on the axis gives the longitudinal spherical aberration. The radius of the circle of least confusion gives the lateral spherical aberration.

#### Methods of minimizing spherical aberration

1. The spherical aberration can be minimized by using stops which reduce the effective lens aperture. The stops can be made to permit either the axial rays or the marginal rays of light.
2. The spherical aberration can be reduced using a spherical designed double convex or concave lens.
3. The Plano-convex lens with its convex surface facing the incident light is used to reduce the spherical aberration.
4. By a suitable combination of convex and concave lenses, the spherical aberration can be reduced to minimum.
5. It can be minimized by using properly designed aplanatic lens.
6. Spherical aberration may be minimized by using two Plano-convex lenses separated by a distance equal to the difference in their focal lengths.

## Condition for minimum spherical aberration of two thin lenses separated by a distance

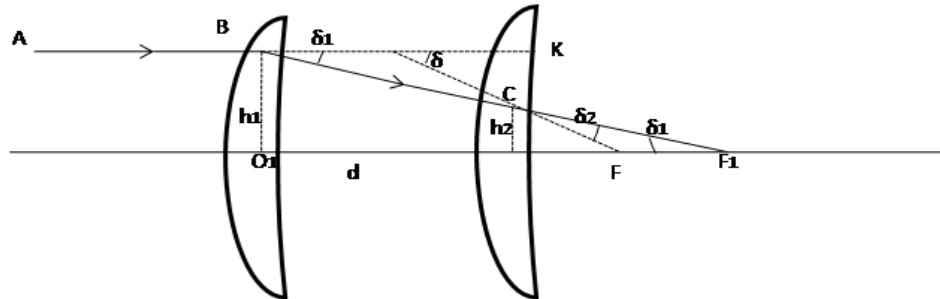


Fig. 1.5

Let two Plano-convex lenses  $L_1$  and  $L_2$  of focal lengths  $f_1$  and  $f_2$  be placed co-axially. They are separated by a distance  $d$ .  $AB$  is an incident ray, strikes the lens  $L_1$  at height  $h_1$  above the principal axis and refracted along  $BC$ .

The deviation  $\delta_1$  produced by the lens  $L_1$  is given by

$$\delta_1 = \frac{h_1}{f_1} \dots \dots \dots (1)$$

The refracted ray  $BC$  is incident at  $C$  at a height  $h_2$  from the axis on lens  $L_2$ .

$$\delta_2 = \frac{h_2}{f_2} \dots \dots \dots (2)$$

For minimizing the spherical aberration,  $\delta_1 = \delta_2$

$$\frac{h_1}{f_1} = \frac{h_2}{f_2}$$

$$\frac{h_1}{h_2} = \frac{f_1}{f_2} \dots \dots \dots (3)$$

From similar triangles  $F_1BO_1$  and  $F_1CO_2$ , we have

$$\frac{O_1B}{O_2C} = \frac{O_1F_1}{O_2F_1} = \frac{h_1}{h_2} = \frac{f_1}{f_1-d} \dots \dots \dots (4) \quad O_1F_1 = f_1 ; O_2F_1 = O_1F_1 - O_1O_2 = f_1 - d$$

Comparing equations (3) and (4), we get

$$\frac{f_1}{f_2} = \frac{f_1}{f_1-d}$$

$$f_2 = f_1 - d$$

$$d = f_1 - f_2$$

Thus, when the lenses are kept separated by a distance  $d = f_1 - f_2$ , there will be minimum spherical aberration for the lens system.

### 1.6.2 Chromatic aberration

The focal length of a lens is given by

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Since  $n$  changes with the colors of light,  $f$  must be different for different colors i.e., the focal length of a lens for violet is less than the focal length for red light. Hence when a point object sending white light is kept on the axis of a lens, the image is formed as linear spectrum instead of single point. This defect in the image –formation is known as the chromatic aberration. It is classified into two types. (a) Longitudinal chromatic aberration (b) Lateral chromatic aberration.

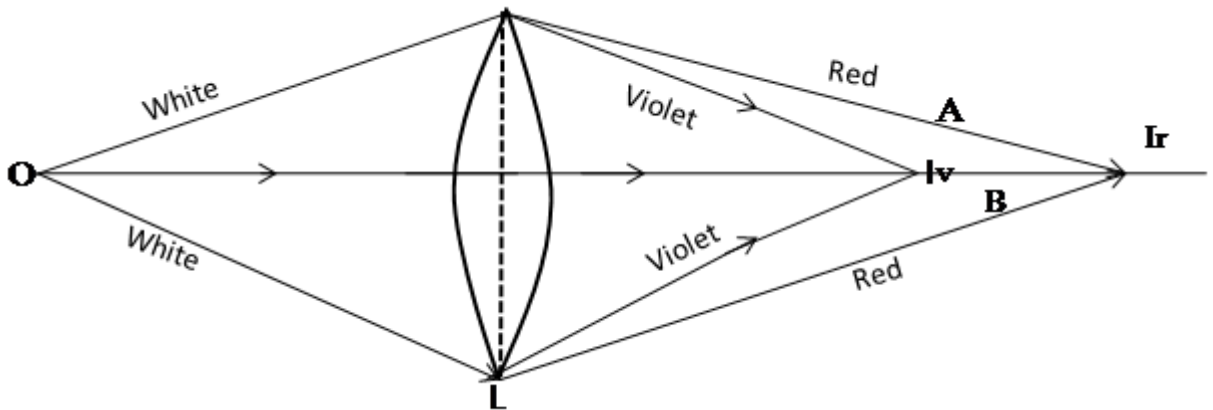


Fig. 1.6

### Expression for chromatic aberration in terms of dispersive power of a lens

According to Lens- maker's formula

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \dots \dots \dots (1)$$

Differentiating equation (1), we get

$$\frac{-1}{f^2} df = dn \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

From equation (1)  $\left( \frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{1}{f(n-1)}$

$$\frac{-1}{f^2} df = \frac{dn}{f(n-1)}$$

$$\frac{-df}{f} = \frac{dn}{(n-1)}$$

$$\frac{-df}{f} = \omega$$

Where  $\omega = \frac{dn}{(n-1)}$  is called the dispersive power ( $\omega$ ) of the material of the lens.

## Removal of chromatic aberration by the use of two lenses

### Case 1: Two lenses in contact

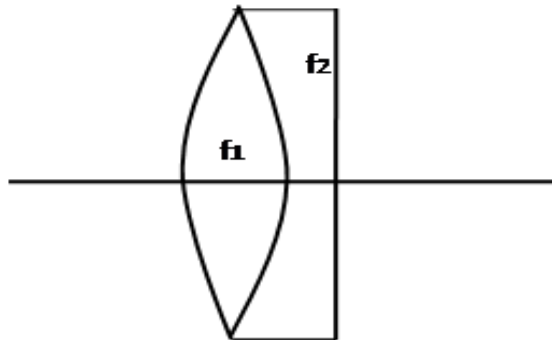


Fig.1.7

Let two thin lenses of focal lengths  $f_1$  and  $f_2$  be kept coaxially in contact with each other. The equivalent focal length  $F$  of the combination is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

Differentiating

$$\frac{-dF}{F^2} = \frac{-df_1}{f_1^2} - \frac{df_2}{f_2^2}$$

To eliminate the chromatic aberration, the  $dF = 0$

$$0 = \frac{-df_1}{f_1^2} - \frac{df_2}{f_2^2}$$

$$\frac{-df_1}{f_1} = \omega_1, \quad -\frac{df_2}{f_2} = \omega_2$$

$$0 = \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2}$$

$$\frac{\omega_1}{f_1} = -\frac{\omega_2}{f_2}$$

$$\frac{\omega_1}{\omega_2} = -\frac{f_1}{f_2}$$

The negative sign shows that in an achromatic doublet arrangement, one lens is convex, the other be concave.

**Case 2: Two lenses separated by a distance**

Consider two lenses of focal lengths  $f_1$  and  $f_2$  be kept coaxially separated by a distance  $D$ .

The equivalent focal length  $F$  of the combination is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{D}{f_1 f_2}$$

Differentiating, we get

$$\frac{-dF}{F^2} = \frac{-df_1}{f_1^2} - \frac{df_2}{f_2^2} - D \left[ \frac{1}{f_1} \left( -\frac{df_2}{f_2^2} \right) + \frac{1}{f_2} \left( \frac{-df_1}{f_1^2} \right) \right]$$

$$\frac{-dF}{F^2} = \frac{-df_1}{f_1^2} - \frac{df_2}{f_2^2} - \frac{D}{f_1 f_2} \left[ \left( -\frac{df_2}{f_2} \right) + \left( \frac{-df_1}{f_1} \right) \right]$$

We know

$$\frac{-df_1}{f_1} = \omega_1, \quad \frac{-df_2}{f_2} = \omega_2$$

$$\frac{-dF}{F^2} = \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} - \frac{D}{f_1 f_2} (\omega_1 + \omega_2)$$

To eliminate chromatic aberration  $dF = 0$

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} - \frac{D}{f_1 f_2} (\omega_1 + \omega_2) = 0$$

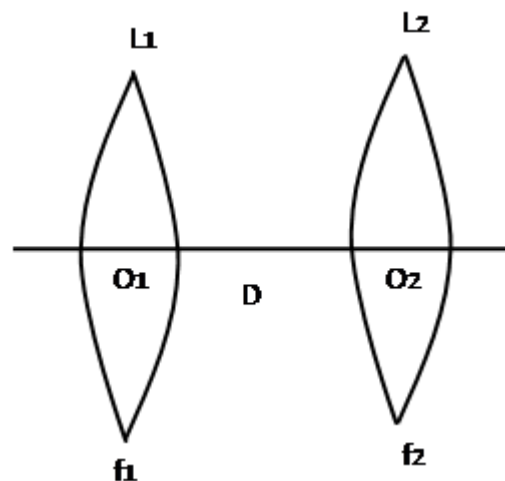


Fig. 1.8

If the lenses are made up of the same material,  $\omega_1 = \omega_2 = \omega$

$$\therefore \frac{\omega}{f_1} + \frac{\omega}{f_2} - \frac{D}{f_1 f_2} 2\omega = 0$$

$$\omega \left( \frac{1}{f_1} + \frac{1}{f_2} - \frac{2D}{f_1 f_2} \right) = 0$$

$$\frac{1}{f_1} + \frac{1}{f_2} - \frac{2D}{f_1 f_2} = 0$$

$$\frac{2D}{f_1 f_2} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$2D = \frac{f_1 f_2}{f_1} + \frac{f_1 f_2}{f_2}$$

$$2D = f_1 + f_2$$

$$D = \frac{f_1 + f_2}{2}$$

That is, the distance between the two coaxial lenses must be equal to half the sum of their focal lengths.

## 1.7 Refraction of light through prism

Consider a prism of refractive index  $n$  and refracting angle  $A$ .  $OP$  is the incident ray;  $PQ$  is the refracted ray.  $QR$  is the emergent ray.  $OPST$  represents the direct ray in the absence of the prism and  $QR$  the emergent ray. The angle between these two rays is known as the angle of deviation 'd'. The angle between the incident ray and normal at  $P$  is known as the angle of incidence 'i'. The angle between the refracted ray and the normal at  $P$  is known as the angle of refraction 'r'. For light passing through a given prism, the angle of deviation decreases as the angle of incidence is increased and reaches the minimum value, thereafter it increases as 'i' is increased. This minimum value of angle of deviation is known as the angle of minimum deviation  $D$ .

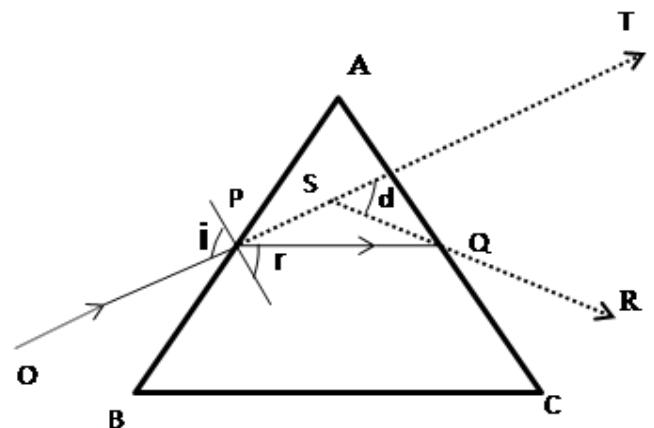


Fig. 1.9

The refractive index of the prism  $n$  is written as

$$n = \frac{\sin \left[ \frac{(A+D)}{2} \right]}{\sin \left[ \frac{A}{2} \right]}$$

For small value of  $A$  and  $D$

$$n = \frac{(A+D)}{A} \quad \text{put } D = d$$

$$n A = A + d$$

$$d = n A - A$$

$$d = (n-1) A$$

This shows the deviation is different for different colors.

## 1.8 Dispersion of light by a prism

When a ray of white light is passed through a glass prism, the ray splits into seven different colors. This is because the refractive index  $n$  depends on the wavelength  $\lambda$  of the light passing through the prism. The dependence of refractive index on the wavelength of light passing through the medium is known as dispersion. The medium is known as dispersive medium.

## 1.9 Dispersive power of the prism

Let  $n_v$  be the refractive index and  $\delta_v$  be the deviation of violet light in a prism. Let  $n_r$  be the refractive index and  $\delta_r$  be the deviation of red light in a prism.

The equation of deviation  $d = (n-1) A \dots \dots \dots (1)$

Deviation of violet  $d_v = (n_v - 1) A \dots \dots \dots (2)$

Deviation of red  $d_r = (n_r - 1) A \dots \dots \dots (3)$

Eqn (2) – eqn (3),

$$(d_v - d_r) = (n_v - n_r) A \dots \dots \dots (4)$$

$\frac{\text{eqn (2)}}{\text{eqn (1)}}$  gives

$$\frac{(d_v - d_r)}{d} = \frac{(n_v - n_r)}{(n - 1)} \dots \dots \dots (5)$$

The quantity  $(d_v - d_r)$  is known as the angular dispersion by the prism.

Dispersive power of a prism is defined as the ratio of angular dispersion to the deviation of mean ray.

ie)  $\omega = \frac{(d_v - d_r)}{d}$

From eqn (5)

$$\omega = \frac{(n_v - n_r)}{(n - 1)}$$

This is the equation for the dispersive power of the prism.

### 1.9.1 Combination of prism to produce deviation without dispersion

When a beam of white light passed through a prism, both deviation and dispersion are produced. A prism that produces deviation without dispersion is called achromatic prism. It is possible to combine two or more prisms to produce deviation of light ray without dispersion. Consider two prisms  $P_1$  and  $P_2$  of angles  $A_1$  and  $A_2$  respectively. Let  $n_{1v}$  and  $n_{1r}$  are the refractive indices of prism  $P_1$  for violet and red rays. Let  $n_{2v}$  and  $n_{2r}$  are the refractive indices of prism  $P_2$  for violet and red rays.

The angular dispersion produced by prism  $P_1$  is

$$= (n_{1v} - n_{1r}) A_1$$

The angular dispersion produced by prism  $P_2$  is

$$= (n_{2v} - n_{2r}) A_2$$

The condition for deviation without dispersion is

$$(n_{1v} - n_{1r})A_1 + (n_{2v} - n_{2r}) A_2 = 0$$

$$(n_{1v} - n_{1r})A_1 = -(n_{2v} - n_{2r}) A_2$$

$$\frac{A_1}{A_2} = - \left( \frac{n_{2v} - n_{2r}}{n_{1v} - n_{1r}} \right)$$

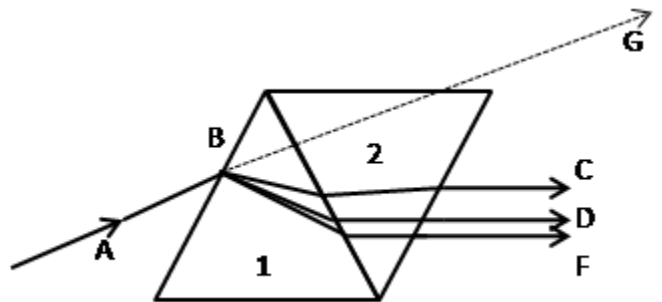


Fig. 1.10

This gives the condition for deviation without dispersion. The negative sign indicates that the refracting angles of the two prisms are in opposite directions.



If two prisms are combined such as to satisfy the above equation, then the combination produces no dispersion of the light passing through it.

Let  $d_1$  be the deviation of the mean ray through the first prism of refractive index  $n_1$ . Let  $d_2$  be the deviation of the mean ray through the second prism of refractive index  $n_2$ .

Then

$$D_1 = (n_1 - 1)A_1$$

$$d_2 = (n_2 - 1)A_2$$

Total deviation produced is  $D = d_1 + d_2$

$$D = (n_1 - 1)A_1 + (n_2 - 1)A_2$$

From the equation  $\frac{A_1}{A_2} = -\left(\frac{n_{2v}-n_{2r}}{n_{1v}-n_{1r}}\right)$

$$A_1 = -A_2 \left(\frac{n_{2v}-n_{2r}}{n_{1v}-n_{1r}}\right)$$

$$\therefore D = (n_1 - 1) \left(-A_2 \left(\frac{n_{2v}-n_{2r}}{n_{1v}-n_{1r}}\right)\right) + (n_2 - 1)A_2$$

$$D = (n_2 - 1)A_2 - A_2 \frac{(n_{2v}-n_{2r})(n_1-1)}{(n_{1v}-n_{1r})}$$

$$D = (n_2 - 1)A_2 \left[1 - \frac{(n_{2v}-n_{2r})(n_1-1)}{(n_{1v}-n_{1r})(n_2-1)}\right]$$

The dispersive power is  $\omega = \frac{(n_v-n_r)}{(n-1)}$

The dispersive power of the first prism is  $\omega_1 = \frac{(n_{1v}-n_{1r})}{(n_1-1)}$

The dispersive power of the second prism is  $\omega_2 = \frac{(n_{2v}-n_{2r})}{(n_2-1)}$

$$\frac{\omega_2}{\omega_1} = \frac{\frac{(n_{2v}-n_{2r})}{(n_2-1)}}{\frac{(n_{1v}-n_{1r})}{(n_1-1)}} = \frac{(n_{2v}-n_{2r})(n_1-1)}{(n_{1v}-n_{1r})(n_2-1)}$$

$$\therefore D = (n_2 - 1)A_2 \left[1 - \frac{\omega_2}{\omega_1}\right]$$

This gives the deviation produced by the combination of prisms for the mean ray.

## 1.9.2 Combination of prism to produce dispersion without deviation

When white light passes through a prism, it suffers deviation and dispersion. Two prisms of different material may be combined so that the resultant deviation is zero. But there will be still dispersion. Let  $A_1$  and  $A_2$  be the angles of two prisms. Let  $n_{1v}$ ,  $n_{1r}$ ,  $n_1$ ,  $n_{2v}$ ,  $n_{2r}$ ,  $n_2$  be the refractive indices of the two materials for the violet, red and mean rays respectively.

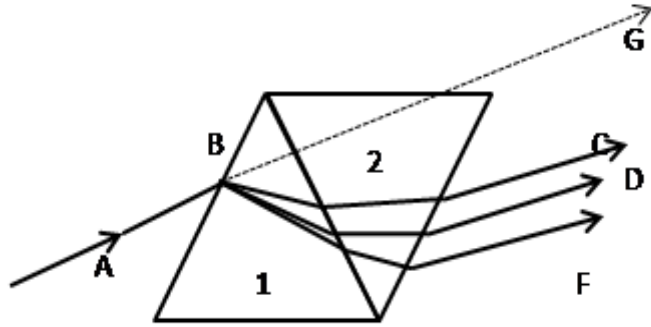


Fig. 1.11

The deviation produced by the two prisms for the mean ray is

$$(n_1 - 1)A_1 + (n_2 - 1)A_2 = 0$$

$$\therefore \frac{A_2}{A_1} = - \frac{(n_1 - 1)}{(n_2 - 1)}$$

The negative sign indicates that the refractive angles of the two prisms are in opposite directions.

The total dispersion produced by the two prisms is

$$\delta = (n_{1v} - n_{1r}) A_1 + (n_{2v} - n_{2r}) A_2$$

$$\text{But, } A_2 = - \frac{(n_1 - 1)}{(n_2 - 1)} A_1$$

$$\therefore \delta = (n_{1v} - n_{1r}) A_1 - \frac{(n_{2v} - n_{2r})(n_1 - 1)}{(n_2 - 1)} A_1$$

$$= (n_1 - 1) A_1 \left[ \frac{(n_{1v} - n_{1r})}{(n_1 - 1)} - \frac{(n_{2v} - n_{2r})}{(n_2 - 1)} \right]$$

$$\delta = -(n_1 - 1) A_1 (\omega_1 - \omega_2)$$

Where  $\omega_1$  and  $\omega_2$  are the dispersive powers of the materials of the two prisms. Thus there is a net dispersion produced, without deviation, in the combination of the prisms.



## UNIT II

### 2.1 Interference

Interference is the phenomenon of superposition of two coherent waves. At some points in the medium, the intensity of light is maximum called constructive interference. At some other points, the intensity is minimum called destructive interference.

#### **Condition for interference**

1. The two interfacing sources must be coherent.
2. The separation between the two sources must be very small.
3. They should emit light of the same frequency or wavelength.
4. The interfering waves must have equal amplitudes.
5. The two interfering waves must be propagated along the same line.

### 2.2 Young's experiment

The experimental arrangement is shown in fig 2.1. A and B are the two narrow slits which are illuminated by the monochromatic light from the source S. Then A and B will act as a coherent sources. i.e. They emit waves of same wavelength and same amplitude. If a screen is placed in the region beyond the slits, a pattern of white and dark interference fringes are seen. Two waves that add constructively give a bright fringe. Two waves that add destructively produce a dark fringe. The white and dark fringes occur alternatively at equal distances.

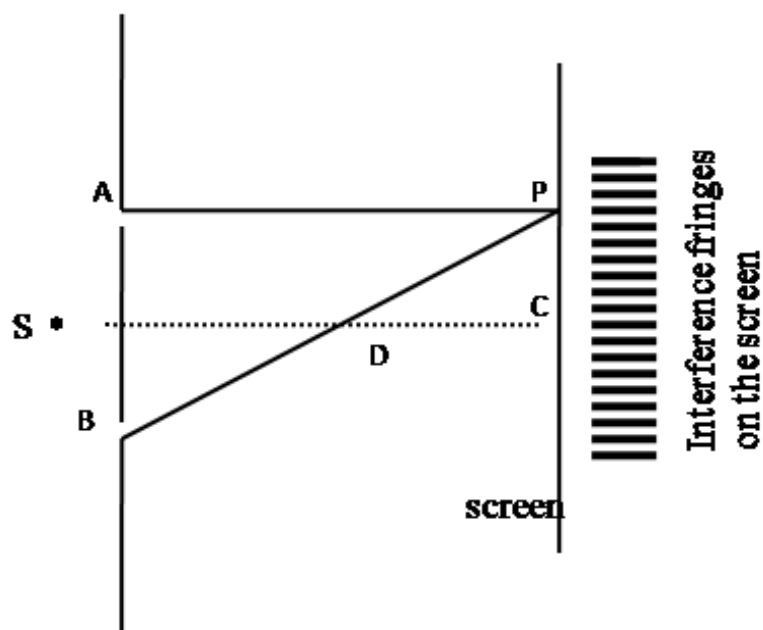


Fig.2.1

## Theory

S is a source of monochromatic light of wavelength  $\lambda$ . A and B are the two narrow slits parallel to S. They act as two virtual coherent sources separated by a distance  $d$ . The point C on the screen is equidistance from A and B. The path difference  $AC - BC = 0$ . Therefore the point C is bright.

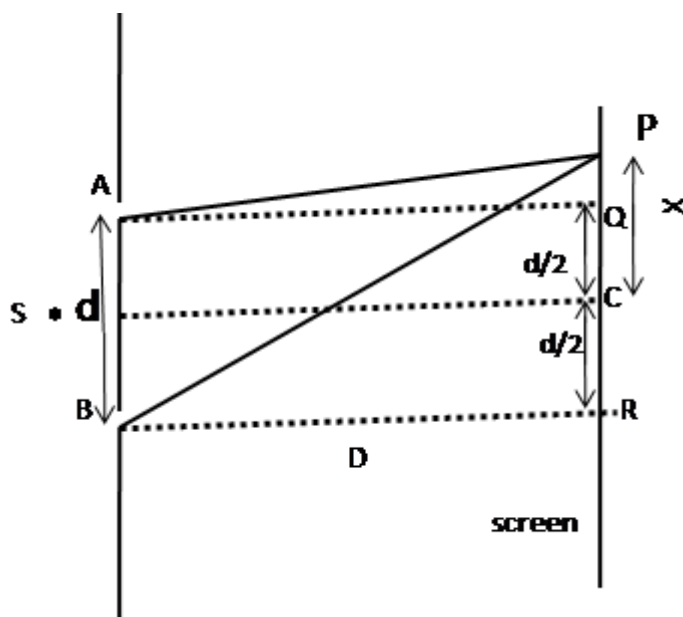


Fig.2.2

Consider a point P at a distance  $x$  from C. Two rays AP and BP starting from A and B reach P and interfere with each other. The path difference between the two waves =  $BP - AP$ .

From figure,

$$PQ = x - \frac{d}{2} \text{ and } PR = x + \frac{d}{2}$$

$$(BP)^2 = D^2 + (x + \frac{d}{2})^2 \text{ and } (AP)^2 = D^2 + (x - \frac{d}{2})^2$$

$$\therefore (BP)^2 - (AP)^2 = 4x\frac{d}{2} = 2xd.$$

$$(BP+AP)(BP -AP) = 2xd$$

$$(BP -AP) = \frac{2xd}{(BP+AP)} = \frac{2xd}{2D} \quad BP \cong AP \cong D$$

$$\therefore \text{The path difference} = (BP -AP) = \frac{xd}{D} \dots\dots\dots(1)$$

Phase difference  $\delta = \frac{2\pi xd}{\lambda D}$ .....(2)

**Case 1:**

when the path difference is  $m\lambda$ , the point P is bright. For  $m^{\text{th}}$  bright fringe

$$\frac{x_m d}{D} = m\lambda. \quad \text{where } m = 0,1,2,3,\dots$$

The position of the  $m^{\text{th}}$  bright fringe is  $x_m = \frac{\lambda D}{d} m$ .

The fringe width  $\beta$  is the distance between two consecutive bright fringes.

$$\therefore \text{Fringe width } \beta = x_{m+1} - x_m = \frac{\lambda D}{d} (m + 1) - \frac{\lambda D}{d} m$$

$$\beta = \frac{\lambda D}{d} \dots\dots\dots(3)$$

**Case 2:**

If the path difference is an odd multiple of  $\lambda/2$ , the point is dark. For  $m^{\text{th}}$  dark fringe ,

$$\frac{x_m d}{D} = (2m + 1)\lambda/2 \quad \text{Where } m=0, 1, 2, 3,\dots$$

The position of the  $m^{\text{th}}$  dark fringe is  $x_m = (2m + 1) \frac{\lambda D}{2d}$ .....(4)

$$\therefore \text{Fringe width } \beta = x_{m+1} - x_m = \frac{\lambda D}{2d} (2m + 3) - \frac{\lambda D}{2d} (2m+1)$$

$$= \frac{\lambda D}{2d} (2m+3-2m-1)$$

$$\beta = \frac{\lambda D}{2d} (2) = \frac{\lambda D}{d}$$

**2.3 Newton’s rings**

A convex lens is placed on the surface of an optically plane glass plate. Then an air film is formed between the lower surface of the lens and upper surface of the lens. If monochromatic light is allowed to fall normally on the lens, a system of alternate bright and dark concentric rings is formed in the air film. These rings are known as Newton’s rings.

### 2.3.1 Determination of wavelength of monochromatic light by Newton's rings method

#### Theory

A convex lens is placed over a plane glass plate. An air film is formed between them. The thickness of the air film is zero at the point of contact O and gradually increases from O outwards. Let a parallel beam of monochromatic light is incident normally on the system. Newton's rings are formed as a result of interference between the light beam reflected from the upper and lower surfaces of the plate.

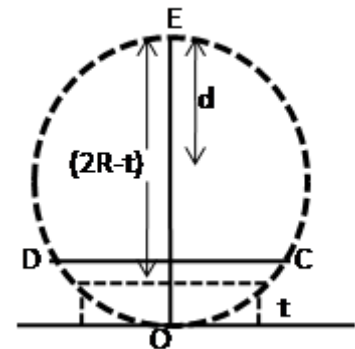


Fig.2.3

Let R be the radius of curvature of the lens and A is the center of curvature. Let there is  $m^{\text{th}}$  dark ring at C. The thickness of the air film at point C is  $OB = t$ . The radius of the  $m^{\text{th}}$  dark ring is  $r_m = DB = BC$

From the geometry of the circle,

$$DB \times BC = EB \times BO$$

$$r_m \times r_m = (2R - t)t$$

$$r_m^2 = (2R - t)t$$

$$T \ll R, \text{ so that } 2R - t \cong 2R$$

$$r_m^2 = 2Rt \dots\dots\dots (1)$$

The condition for a dark ring is  $2nt = m\lambda$  where  $m = 0, 1, 2, 3, \dots$

$$t = \frac{m\lambda}{2n} \dots\dots\dots (2)$$

Substituting eqn (2) in eqn (1), we get,

$$r_m^2 = \frac{2Rm\lambda}{2n} = \frac{Rm\lambda}{n}$$

$$\text{Radius of the } m^{\text{th}} \text{ dark ring} = r_m = \sqrt{\frac{Rm\lambda}{n}} \dots\dots\dots (3)$$

The condition for a bright ring is  $2nt = (2m-1) \lambda/2$  where  $m=0, 1, 2, 3, \dots$

$$t = \frac{(2m-1)\lambda/2}{2n} \dots\dots\dots(4)$$

Substituting eqn (4) in eqn (1), we get,

$$r_m^2 = \frac{2R(2m-1)\lambda/2}{2n} = \frac{(2m-1)R\lambda}{2n}$$

Radius of the  $m^{\text{th}}$  bright ring  $r_m = \sqrt{\frac{(2m-1)R\lambda}{2n}} \dots\dots\dots(5)$

The diameters of  $m^{\text{th}}$  dark ring and  $m^{\text{th}}$  bright ring are

$$D_m(\text{dark}) = \sqrt{\frac{4Rm\lambda}{n}}$$

$$D_m(\text{bright}) = \sqrt{\frac{2(2m-1)R\lambda}{n}}$$

Let  $D_m$  be the diameter of the  $m^{\text{th}}$  dark ring and  $D_p$  be the diameter of the  $p^{\text{th}}$  dark ring.

$$D_m^2 = \frac{4Rm\lambda}{n} \quad \text{For air refractive index } n=1$$

$$D_m^2 = 4Rm\lambda$$

$$D_p^2 = 4Rp\lambda$$

$$D_m^2 - D_p^2 = 4Rm\lambda - 4Rp\lambda$$

$$D_m^2 - D_p^2 = 4R\lambda(m - p)$$

$$\lambda = \frac{D_m^2 - D_p^2}{4R(m-p)}$$

## Experiment

The figure 2.4 shows an experimental arrangement. S is the source of monochromatic light. Parallel rays of light from the source fall on a glass plate kept inclined at  $45^\circ$  and are partly reflected from it. This reflected beam falls normally on the lens L placed on the glass plate PQ. Interference occurs between the rays reflected from the upper and lower surfaces of the



film and produce circular interference fringes called Newton's rings. The interference rings are viewed through microscope M.

The microscope is adjusted so that the cross wire coincides with the 18<sup>th</sup> dark ring on the left and the microscope readings is notes in the horizontal scale. Similarly, readings are taken at 15, 12, 9, 6, 3<sup>rd</sup> dark rings on the left side. Readings are taken corresponding to 3, 6, 9, 12, 15, 18<sup>th</sup> dark rings on the right side also. From these readings the diameter of various dark rings are found. Let  $D_m$  be the diameter of the  $m^{\text{th}}$  dark ring and  $D_p$  be the diameter of the  $p^{\text{th}}$  dark ring.

The radius of the  $m^{\text{th}}$  dark ring  $r_m^2 = Rm\lambda$

$$\left(\frac{D_m}{2}\right)^2 = mR\lambda$$

$$D_m^2 = 4Rm\lambda$$

Similarly,

$$D_p^2 = 4Rp\lambda$$

$$D_m^2 - D_p^2 = 4Rm\lambda - 4Rp\lambda$$

$$D_m^2 - D_p^2 = 4R\lambda(m - p)$$

$$\lambda = \frac{D_m^2 - D_p^2}{4R(m - p)}$$

This gives the wavelength of the monochromatic light.

## 2.4 Air wedge

Two perfectly plane glass plates inclined at a very small angle with each other forms a thin film of air. This film is in wedge shape and the arrangement of the two plates is known as air wedge.

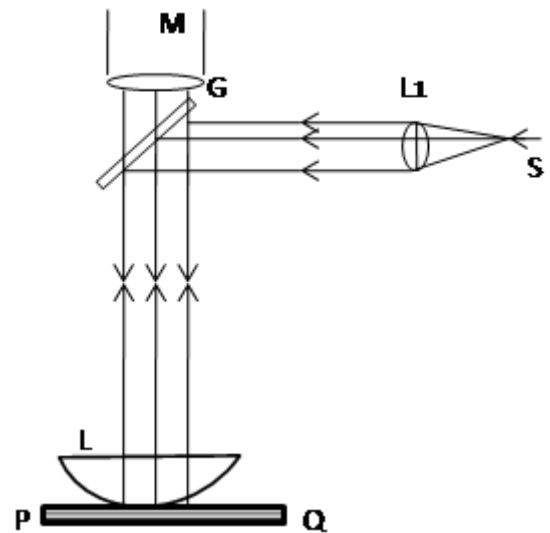


Fig. 2.4

### 2.4.1 Determination of diameter of a thin wire using air wedge method Theory

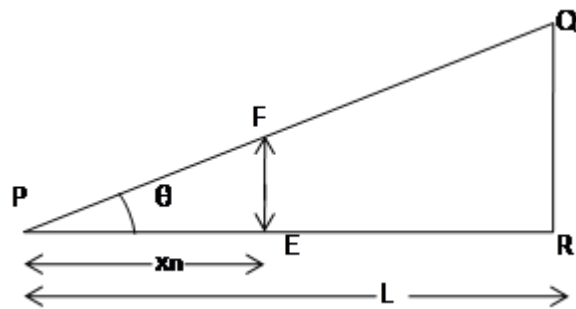


Fig.2.5

Let PQ and PR are the two optically plane glass plates. The glass plates are tied together at one end and separated by a thin wire at the other end. Let  $\theta$  be the angle between the plates. Air film in wedge shape is formed in the space between the plates. When the air film is illuminated normally with monochromatic light interference occurs between the rays reflected at the upper and lower surfaces of the air film. We can see the interference fringes of equal width using a microscope.

Let us consider the  $n^{\text{th}}$  dark fringe at a distance  $x_n$  from P. let  $t$  be the thickness of the air wedge at  $n^{\text{th}}$  dark fringe.

The condition for  $n^{\text{th}}$  dark fringe is  $2t = n\lambda$ .....(1)

Let  $L$  be the distance between P and R and  $d$  is the distance between the two plates at R. from similar triangles  $\Delta PEF$  and  $\Delta PRQ$

$$\frac{x_n}{L} = \frac{t}{d}$$

$$t = \frac{x_n d}{L} \dots\dots\dots(2)$$

Substituting eqn (2) in eqn (1) we get,

$$2 \frac{x_n d}{L} = n\lambda$$

$$x_n = \frac{n\lambda L}{2d} \dots\dots\dots(3)$$

For  $(n+1)^{\text{th}}$  dark fringe which is at a distance  $x_{n+1}$

$$x_{n+1} = \frac{L}{2d}(n + 1)\lambda$$

The distance between two dark fringes is called fringe width  $\beta$

$$\beta = x_{n+1} - x_n = \frac{L}{2d}(n + 1)\lambda - \frac{L}{2d}n\lambda$$

$$\beta = \frac{L\lambda}{2d}$$

$$d = \frac{L\lambda}{2\beta}$$

## Experiment

An air wedge is formed with two glass plates tied along one end and separated by a thin wire at the other end. The air wedge is mounted on the bed plate of a travelling microscope. Monochromatic light is reflected vertically downwards on the wedge by the inclined glass plate kept above the wedge. Straight line fringes are formed and viewed through the microscope. The width of twenty fringes is measured using the microscope and from that, the fringe width  $\beta$  is found out. The length of the wedge is measured as  $L$ . knowing  $\lambda$ , the diameter  $d$  of the wire is calculated using the formula  $d = \frac{L\lambda}{2\beta}$ .

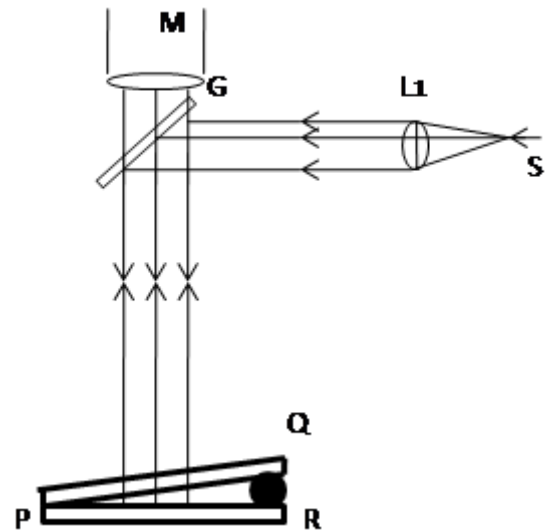


Fig. 2.6



## UNIT III

### 3.1 Diffraction

The phenomenon of bending of waves around the edge of the obstacle is called diffraction.

### 3.2 Types of diffraction

1. Fresnel diffraction
2. Fraunhofer diffraction

#### 3.2.1 Fresnel diffraction

In the Fresnel diffraction, the source and the screen are at finite distances from the obstacle. No lenses are used to focus the rays. In such case the wave front undergoing diffraction is either

Spherical or cylindrical.

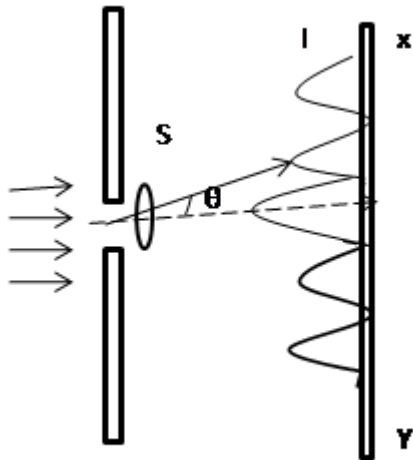


Fig. 3.1 Fresnel diffraction

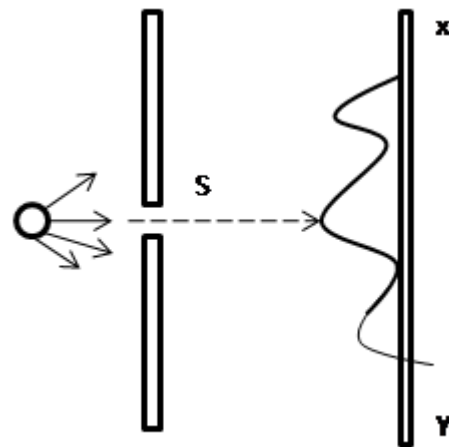


Fig.3.2 Fraunhofer diffraction

#### 3.2.2 Fraunhofer diffraction

In the Fraunhofer diffraction, the source and the screen are at infinite distances from the obstacle. The rays reaching a point are parallel and are brought to focus by a converging lens. In this case the wave front undergoing diffraction is a plane wave front.

### 3.3 Theory of plane transmission grating

The plane transmission grating is a plane sheet of transparent material on which opaque rulings are made with a diamond point.

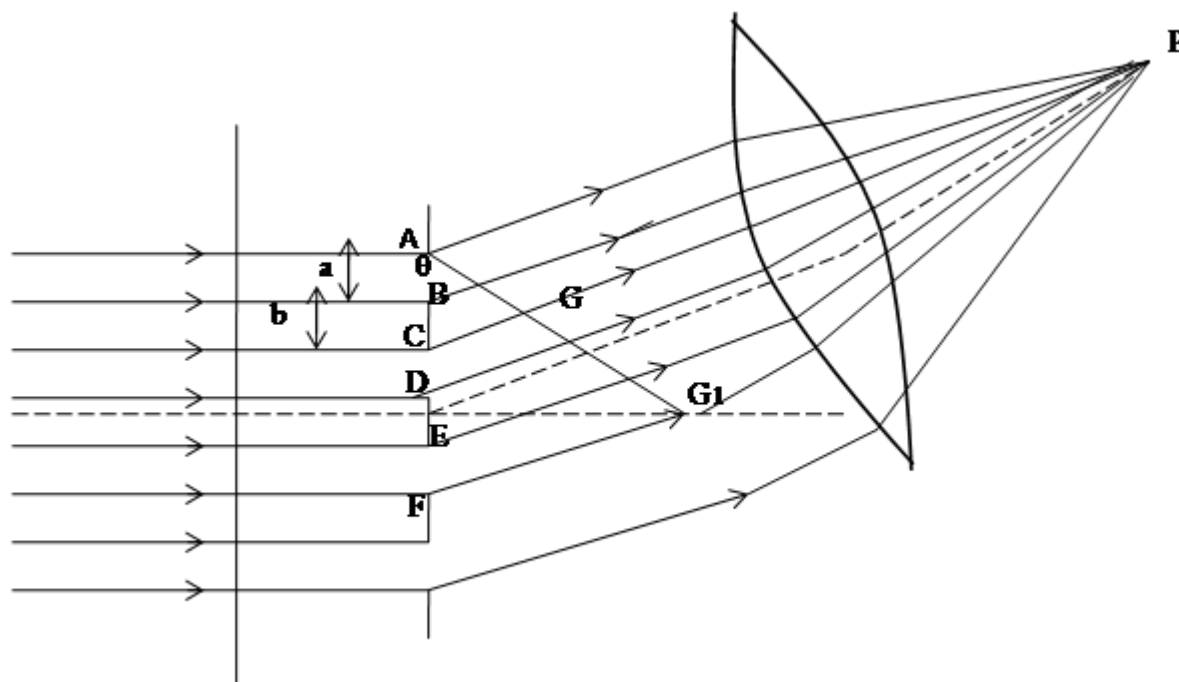


Fig. 3.3

Consider a plane transmission grating. AB, CD, EF, etc are the transparent slits of width  $a$ . BC, DE, etc represent the rulings of width  $b$ . Then the distance  $(a+b)$  is called the grating element. When a parallel beam of light incident normally on the grating, the points in the Slits AB, CD, EFF, etc act as secondary source of light and give secondary wavelets which spread in all directions on the other side of the grating. Therefore, the rays are diffracted in each slit in all directions. The rays diffracted at an angle  $\theta$  with the direction of the incident ray are focused at the point P using the convex lens L. Draw AG perpendicular to the direction of the diffracted light. Then CG is the path difference between the rays diffracted from the two corresponding points A and C at an angle  $\theta$ . The path difference  $CG = AC \sin\theta = (a+b) \sin\theta$

The point will be bright when the path difference  $(a+b) \sin\theta = m\lambda$

For  $m = 0$ ,  $(a+b) \sin\theta = 0$  ie. All points in the direction of the incident light will be bright. This is the zero order diffracted image.

For  $m = 1$ ,  $(a+b) \sin\theta_1 = \lambda$ ; the first order image is obtained. Similarly the second order bright image is obtained when  $(a+b) \sin\theta_2 = 2\lambda$ . The different order bright images are obtained on the both sides of the direct ray.

In general  $(a+b) \sin\theta = m \lambda$  is the condition for brightness.

$$\sin\theta = \frac{m\lambda}{a+b} = Nm\lambda$$

Where  $N = \frac{1}{a+b}$  --- the number of grating elements or lines per unit length of the grating.

### 3.4 Experiment to determine the wavelength of light using plane transmission

Adjust a spectrometer for work. Illuminate the slit with the source whose wavelength is required. Take the direct reading of the telescope. Rotate the prism table until the image of the slit reflected by the grating coincides with the cross-wire. Rotate the prism table by  $45^\circ$  in the proper direction so that the plane of the grating is normal from the collimator and fix the prism table. In this position, the grating is normal to the incident light.

Bring the telescope to make line with the collimator. Move the telescope towards the left side so that the image of the first order coincides with the vertical cross wire of the telescope and note the reading as  $R_1$ . Then move the telescope on the right side of direct ray so that the first order image on the right side coincides with the cross wire and note as  $R_2$ . The difference between the readings  $R_1$  and  $R_2$  gives  $2\theta$  from which the angle of diffraction  $\theta$  can be found out.

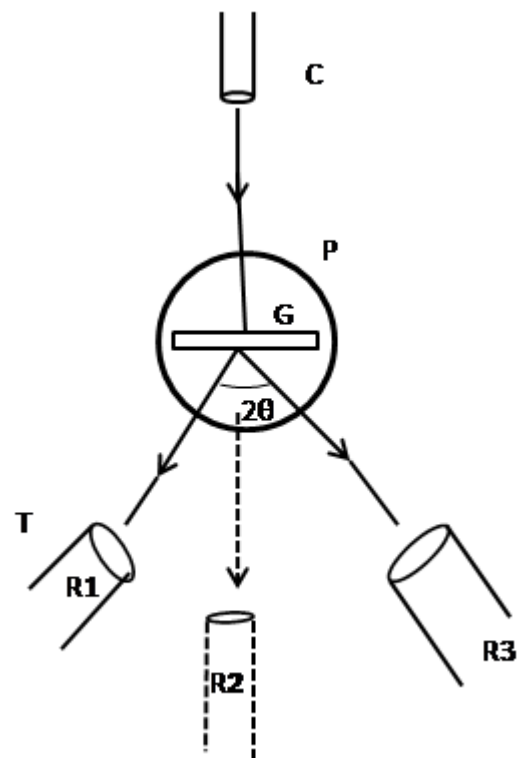


Fig.3.4

The number of lines (N) per unit length on the grating is known from the data given by the manufacturer. Knowing the angle of diffraction  $\theta$  for a given order of diffraction  $m$  and the number of lines  $N$  per unit length, the wavelength of the light can be calculated using the relation

$$\sin\theta = Nm\lambda$$

$$\lambda = \frac{\sin\theta}{Nm}$$

### 3.5 Dispersive power of grating

Dispersive power of a grating is defined as the ratio of the difference in the angle of diffraction of any two neighboring spectral lines to the difference in wavelength between the two spectral lines. For a plane transmission grating, we have

$$(a+b) \sin\theta = m\lambda \dots\dots\dots(1)$$

Differentiating the equation (1) with respect to  $\theta$  and  $\lambda$ , we get

$$(a+b) \cos\theta d\theta = m d\lambda$$

$$\frac{d\theta}{d\lambda} = \frac{m}{(a+b) \cos\theta} \text{ is the dispersive power of the grating.}$$

The above equation shows that:

1. The dispersive power is directly proportional to  $m$ , the order of the spectrum.
2. The dispersive power is inversely proportional to the grating element  $(a+b)$ .  $\frac{1}{(a+b)} = N$

$$\therefore \frac{d\theta}{d\lambda} \propto N$$

3. The dispersive power is inversely proportional to  $\cos\theta$ .

For small value of  $\theta$ ,  $\cos\theta$  is constant and the influence of the factor  $\cos\theta$  can be neglected. Then  $d\theta \propto d\lambda$  for a given order. That is, the angular dispersion of two spectral lines in a particular order is directly proportional to the difference in wavelength of the spectral lines. A spectrum of this type is called a normal spectrum.

### 3.6 Resolving power of a plane transmission grating

The resolving power of a grating is defined as the ratio of the wavelength  $\lambda$  of a spectral line to the smallest difference in the wavelength  $d\lambda$ , between this line and a neighboring line



which can just be resolved. It is also defined as the ability to show two neighboring lines in a spectrum as separate.

Resolving power of a grating =  $\lambda/d\lambda$

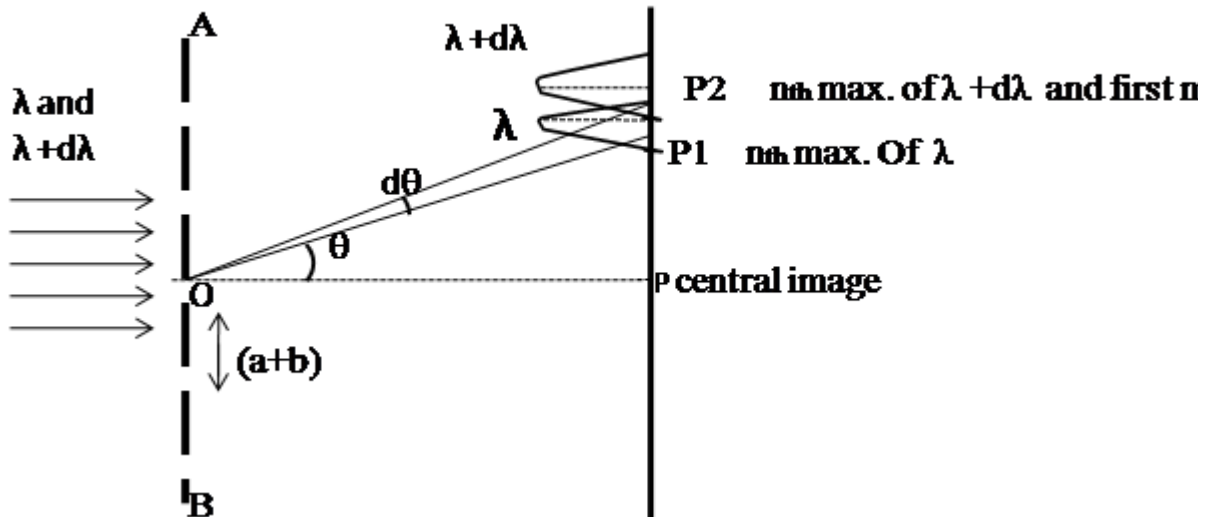


Fig.3.5

In this figure, light of two wavelengths  $\lambda$  and  $(\lambda+d\lambda)$  is incident normally on the surface of the grating AB. The light of each wavelength will form a separate diffraction pattern of the slit.  $P_1$  is the  $n^{\text{th}}$  primary maximum of a spectral lines of wavelength  $\lambda$  at an angle of diffraction  $\theta$ .

$$(a+b) \sin\theta = m\lambda \quad \dots\dots\dots(1)$$

Here  $(a+b)$  is the grating element.

$P_2$  is the  $n^{\text{th}}$  primary maximum of a second spectral line of wavelength  $(\lambda+d\lambda)$  at an angle of diffraction  $(\theta+d\theta)$ , then

$$(a+b) \sin(\theta+d\theta) = m(\lambda+d\lambda) \quad \dots\dots\dots(2)$$

According to Rayleigh, the two spectral lines will appear just resolved if the principal maximum due  $(\lambda+d\lambda)$  falls on the first minimum of  $\lambda$  or vice versa. Thus, the two lines will appear just resolved if the angle of diffraction  $(\theta+d\theta)$  also corresponds to the direction of first secondary minimum after the  $n^{\text{th}}$  primary maximum at  $p_1$  corresponding to wavelength  $\lambda$ . This is possible if the extra path difference introduced is  $\lambda/N$ .

$$\therefore (a+b) \sin (\theta + d\theta ) = m \lambda + \lambda/N \dots \dots \dots (3)$$

Equating the right hand sides of the eqns (2) and (3)

$$m(\lambda+d\lambda) = m \lambda + \lambda/N$$

$$md\lambda = \lambda/N$$

$$\frac{\lambda}{d\lambda} = mN . \text{ This quantity measures the resolving power of grating.}$$

The resolving power increases with (i) the order m of the spectrum and (ii) the total number of lines N on the grating.

### 3.7 Zone plate

A zone plate is a specially constructed screen such that light is obstructed from every alternate zone.

#### Construction

To construct a zone plate, concentric circles with radii proportional to the square roots of natural numbers are drawn on the white paper. The odd numbered zones are painted black and reduced photograph is taken on a thin glass plate. In the negative of the photograph, the odd zones, which were painted black, appear transparent and the even zones appear black.



Fig.3.6

#### Theory

Let AB represent the section of the zone plate perpendicular to the plane of paper. S is a point source of light placed on the axis at a distance ‘a’ from the centre O of the zone plate. P is a point on the screen at a distance ‘b’ from O.  $OM_1(r_1)$  ,  $OM_2(r_2)$  ,  $OM_3(r_3)$ , etc., are the radii of the 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup> etc., half period zones. The position of the screen is such that from one zone to the next, there is an increasing path difference of  $\lambda/2$ .

From figure,

$$SO + OP = a+b$$

$$SM_1 + M_1P = a+b + \lambda/2$$

$$SM_2 + M_2P = a+b + 2\lambda/2 \quad \text{and so on.}$$

From  $\Delta SM_1O$

$$(SM_1)^2 = (SO)^2 + (OM_1)^2$$

$$SM_1 = ((SO)^2 + (OM_1)^2)^{\frac{1}{2}}$$

$$= (a^2 + r_1^2)^{\frac{1}{2}}$$

From  $\Delta OM_1P$

$$(M_1P)^2 = (OP)^2 + (OM_1)^2$$

$$M_1P = ((OP)^2 + (OM_1)^2)^{\frac{1}{2}}$$

$$= (b^2 + r_1^2)^{\frac{1}{2}}$$

Substituting the values of  $SM_1$  and  $M_1P$  in equation (1), we get

$$(a^2 + r_1^2)^{\frac{1}{2}} + (b^2 + r_1^2)^{\frac{1}{2}} = a + b + \frac{\lambda}{2}$$

$$a \left[ \left( 1 + \left( \frac{r_1^2}{a^2} \right)^{\frac{1}{2}} \right) \right] + b \left[ \left( 1 + \left( \frac{r_1^2}{b^2} \right)^{\frac{1}{2}} \right) \right] = a + b + \frac{\lambda}{2}$$

$$a \left[ 1 + \frac{r_1^2}{2a^2} + \dots \right] + b \left[ 1 + \frac{r_1^2}{2b^2} + \dots \right] = a + b + \frac{\lambda}{2}$$

$$a \left[ 1 + \frac{r_1^2}{2a^2} \right] + b \left[ 1 + \frac{r_1^2}{2b^2} \right] = a + b + \frac{\lambda}{2}$$

neglected

$$a + \frac{r_1^2}{2a} + b + \frac{r_1^2}{2b} = a + b + \frac{\lambda}{2}$$

$$\frac{r_1^2}{2} \left[ \frac{1}{a} + \frac{1}{b} \right] = \frac{\lambda}{2}$$

$$r_1^2 \left[ \frac{1}{a} + \frac{1}{b} \right] = \lambda$$

Similarly for  $r_n$  i.e., the radius of the  $n^{\text{th}}$  zone, the relation can be written as

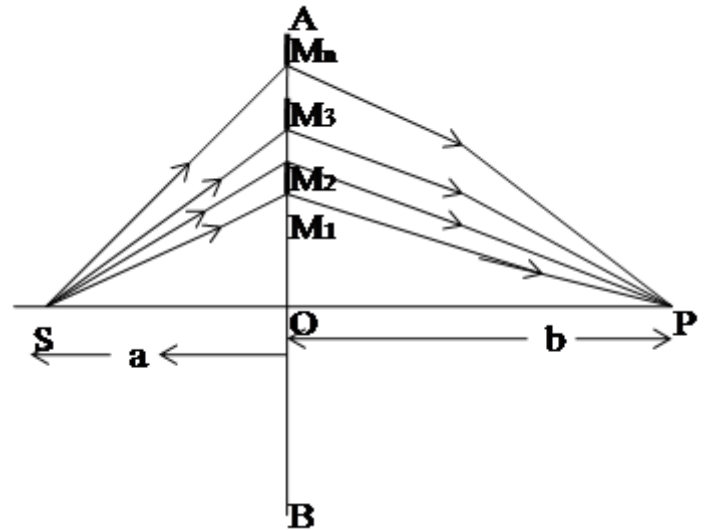


Fig.3.7

$r_n \ll a \therefore$  higher terms will be

$$r_n^2 \left[ \frac{1}{a} + \frac{1}{b} \right] = n\lambda$$

Applying the sign conversion,

$$\frac{1}{b} - \frac{1}{a} = \frac{n\lambda}{r_n^2} = \frac{1}{f_n}$$

$$f_n = \frac{r_n^2}{n\lambda}$$

This equation is similar to the lens equation  $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ . Thus, a zone plate acts as a converging lens.

## UNIT IV

### 4.1 DOUBLE REFRACTION

When a ray of unpolarised light is passed through a calcite crystal, it is split up into two refracted rays. One of the refracted rays follows the ordinary laws of refraction. Hence it is called as ordinary ray. The other refracted ray does not obey the law of refraction. It is called as extraordinary ray. Therefore if an object is viewed through such a crystal, two images of the object are observed. One image corresponds to o-ray and the other to E- ray. This phenomenon is called as double refraction.

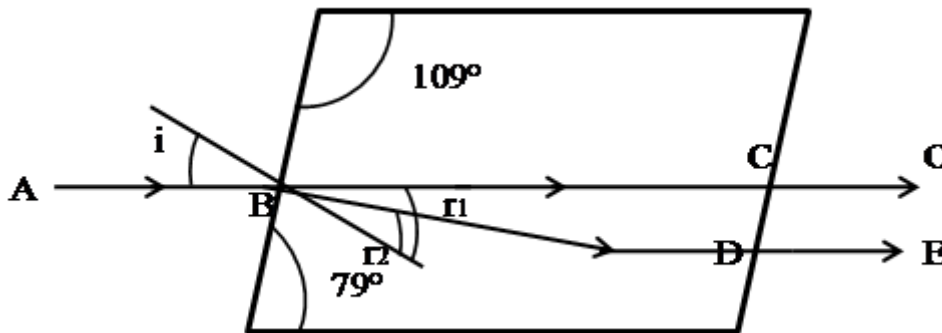


Fig. 4.1

Let a ray AB incident on a calcite crystal at an angle of incidence 'i'. It is split in to two rays (O and E) inside the crystal. The O-ray travelling along BC makes an angle of refraction  $r_1$ . The E-ray travelling along BD makes an angle of refraction  $r_2$ . These two rays emerge out along CO and DE.

The refractive index of ordinary ray  $n_o = \frac{\sin i}{\sin r_1}$ .

The refractive index of extra-ordinary ray  $n_e = \frac{\sin i}{\sin r_2}$ .

Refractive index of O-ray  $n_o$  is constant, since it obeys ordinary laws of refraction. Refractive index of e-ray varies with the angle of incidence  $i$ . the o-ray has same velocity in all directions. The E-ray has different velocities in different directions. Along the optic axis , the velocity of the E- ray is the same as that of the O-ray.

## 4.2 Nicol prism

It is an optical device made from a calcite crystal. It is used for producing and analyzing plane polarized light based on the principle of double refraction. When a ray of unpolarised light is passed through a calcite crystal, it is split into o-ray and the E-ray. Both are plane polarized. In Nicol prism, O-ray is eliminated by total internal reflection. Hence only the plane polarized E-ray is transmitted through the prism.

### Construction

A calcite crystal whose length is 3 times its breadth is taken. Its ends are ground such that the angles in the principal section become  $68^\circ$  and  $112^\circ$  instead of  $71^\circ$  and  $109^\circ$ . Then the crystal is cut into two along the plane perpendicular to both the principal section and the end faces. The two parts of the crystal are then cemented together with Canada balsam which is a transparent liquid of refractive index 1.55 for sodium light. The crystal is then enclosed in a tube blackened inside.

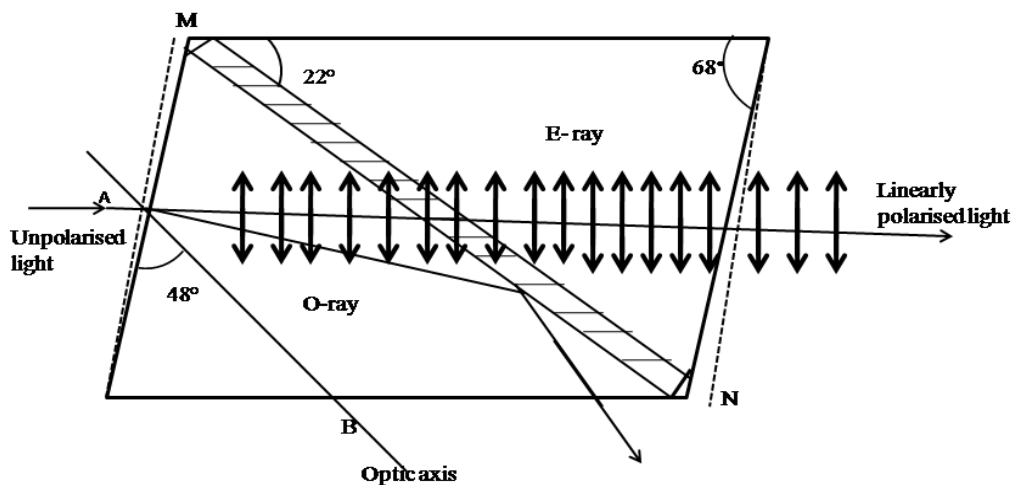


Fig.4.2

Unpolarised light is made to fall on the crystal at an angle of about  $15^\circ$  as shown in figure. The ray gets double refraction inside the crystal and split in to O-ray and E-ray. The cutting is such that the O-ray is incident on the Canada balsom surface at an angle greater than the critical angle and is totally reflected. The face where the O-ray is incident is blackened so that the O-ray is completely absorbed. The E-ray travels from an optically rarer medium to a denser medium. Therefore it is transmitted through the prism. This E-ray is plane polarized

and has vibrations in the principal section parallel to the shorter diagonal of the end face of the crystal. Thus we get a single ray of plane polarized light. So the Nicol prism can be used as a polariser.

### 4.3 Quarter wave plate

A plate of doubly refracting uniaxial crystal whose refracting faces are cut parallel to the optic axis and whose thickness is such as produce a phase difference of  $\pi/2$  or path difference of  $\lambda/4$  between O-ray and E-ray is called a quarter wave plate. Let  $t$  be the thickness of the plate.  $\mu_o$  and  $\mu_e$  are the refractive indices of the O-ray and E-ray respectively.

The path difference is given by  $\delta = \mu_o t - \mu_e t = (\mu_o - \mu_e)t$

For quarter wave plate  $\delta = \lambda/4$

$$\therefore \frac{\lambda}{4} = (\mu_o - \mu_e)t$$

$$t = \frac{\lambda}{4(\mu_o - \mu_e)}$$

For positive crystal such as quartz  $\mu_e > \mu_o$ . For negative crystal such as quartz  $\mu_e < \mu_o$ . The quarter wave plate is used for producing circularly and elliptically polarized light.

### 4.4 Half wave plate

A plate of doubly refracting uniaxial crystal whose refracting faces are cut parallel to the optic axis and whose thickness is such as produce a phase difference of  $\pi$  or path difference of  $\lambda/2$  between O-ray and E-ray is called a Half wave plate. Let  $t$  be the thickness of the plate.  $\mu_o$  and  $\mu_e$  are the refractive indices of the O-ray and E-ray respectively.

The path difference is given by  $\delta = \mu_o t - \mu_e t = (\mu_o - \mu_e)t$

For Half wave plate  $\delta = \lambda/2$

$$\therefore \frac{\lambda}{2} = (\mu_o - \mu_e)t$$

$$t = \frac{\lambda}{2(\mu_o - \mu_e)}$$

Half wave plate is used in polarimeters as half –shade devices.

## 4.5 Production of plane, elliptically and circularly polarized light

### 1. *Plane polarized light*

A plane polarized light can be produced by passing a beam of unpolarised monochromatic light through a Nicol prism. Nicol prism splits the beam into O-ray and E-ray. The O-ray is removed by total internal refraction at the Canada balsam. The E-ray emerges out of the Nicol prism. It is plane polarized with vibrations parallel to the principal plane of the Nicol prism.

### 2. *Elliptically polarized light*

A plane polarized light is allowed to fall normally on a quarter wave plates, such that the vibrations in the plane polarized incident light make an angle  $\theta$  ( $\theta \neq 0, 45^\circ, 90^\circ$ ) with the optical axis of the plate. The plane polarized light on entering the quarter wave plate is split up into O and E rays having unequal amplitudes. On emergence from the quarter wave plate, there is a phase difference of  $\pi/2$  between the two components. They combine to form elliptically polarized light.

### 3. *Circularly polarized light*

Circularly polarized light is produced if the amplitudes of the O and E-rays are equal and there is a phase difference of  $\pi/2$  or a path difference of  $\lambda/4$  between them. A parallel beam of plane polarized light is allowed to fall normally on a quarter wave plate; such that the vibrations of the incident plane polarized light makes an angle of  $45^\circ$  with the optic axis of the plate. The light emerging from the quarter wave plate is circularly polarized.

## 4.6 Detection of plane, circularly, and elliptically polarized light

### 1. *Plane polarized light*

The beam is allowed to pass through a Nicol prism. The Nicol prism is rotated gradually about the direction of propagation of light. If the intensity of the light varies between a maximum and zero, the light is plane polarized light.

### 2. *Circularly polarized light*

The circularly polarized light, when observed through a rotating Nicol prism shows no variation in intensity. For unpolarised light also, there is no change in intensity. To distinguish between circularly polarized light and unpolarised light, light is passed through a quarter wave plate before passing through the analyzing Nicol. The quarter wave plate converts the circularly polarized light in to plane polarized light which



when observed through the rotating Nicol shows intensity variation between maximum and zero. But in the case of unpolarised light intensity still remains constant.

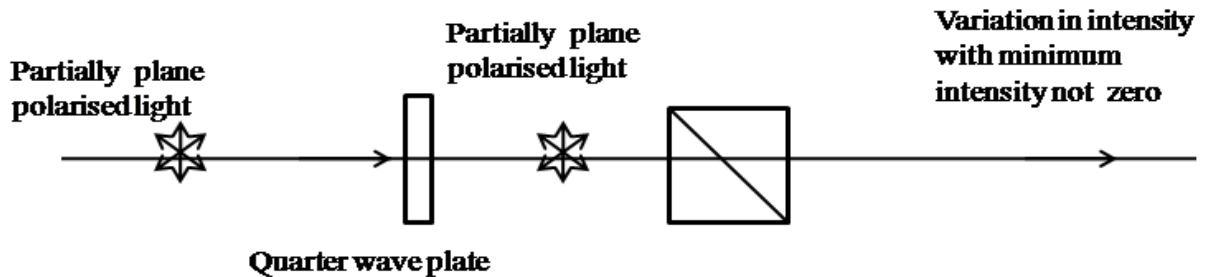


Fig.4.3

### 3. *Elliptically polarized light*

When elliptically polarized light is passed through a rotating Nicol prism, the intensity varies between a maximum and minimum which is not equal to zero. For partially plane polarized light also, the intensity is similar to that of elliptically polarized light. So to distinguish between elliptically light and partially plane polarized light, the beam of light is passed through a quarter wave plate before passing through the analyzing Nicol. The quarter wave plate converts the elliptically polarized light into plane polarized light which when observed through the rotating Nicol shows intensity variation between maximum and zero. But in the case of partially plane polarized light, the intensity still varies between the maximum and minimum which is not equal to zero.

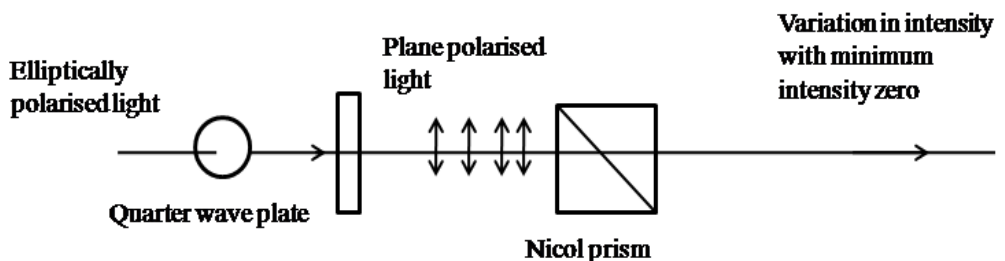


Fig.4.4

## 4.7 Analysis of polarized light

A device which examines whether the light is plane polarized or not is called as an analyser. Nicol prism is used as an analyser.

### Procedure

#### **Step 1:**

The given beam of light is passed through a Nicol prism. The Nicol prism is rotated about the direction of propagation of light. The changes in intensity are noted. There are three possibilities;

1. The intensity does not vary at all. Then the given beam of light is either unpolarised or circularly polarized.
2. The intensity shows variation –two maxima and two minima during one rotation but not be zero. Then the given beam of light is either partially plane polarized or elliptically polarized.
3. The intensity shows variations and completely extinguished twice in each rotation. Then the given beam of light is completely plane polarized.

#### **Step 2:**

To distinguish between circularly polarized light and unpolarised light, the given beam of light is first passed through a quarter wave plate, and then passed through a rotating Nicol.

1. If the intensity varies with zero minimum, the given light is circularly polarized.
2. If there is no variation in intensity, the given beam of light is unpolarised.

#### **Step 3:**

To distinguish between elliptically polarized and partially plane polarized light, the Nicol prism is first adjusted for maximum intensity. Then a quarter wave plate is inserted between the given light and Nicol such that light falls normally on it and its optical axis is parallel to the principal section of the Nicol.

1. On rotating the Nicol prism, if the intensity varies with zero minimum, the given light is elliptically polarized.

- On rotating the Nicol, if the emergent light shows variation in intensity with a non-zero minimum, the given beam of light is partially plane polarized.

### 4.8 Laurent's Half - shade polarimeter

A polarimeter is an instrument used for determining the optical rotation of solutions.

#### Construction

It consists of a monochromatic source S. the light from the source is rendered parallel by a convex lens L and falls on the polarizing Nicol prism P which converts it in to plane polarized light. This light passes through a half shade device H and through the tube T containing the solution. The transmitted light passes through the analyser A which can be rotated about the axis of the tube and its position can be read on a circular scale. The light coming from the analyser is observed through a telescope G.

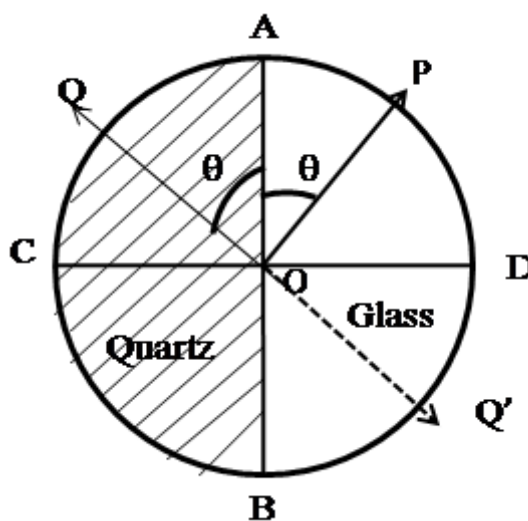


Fig. 4.5

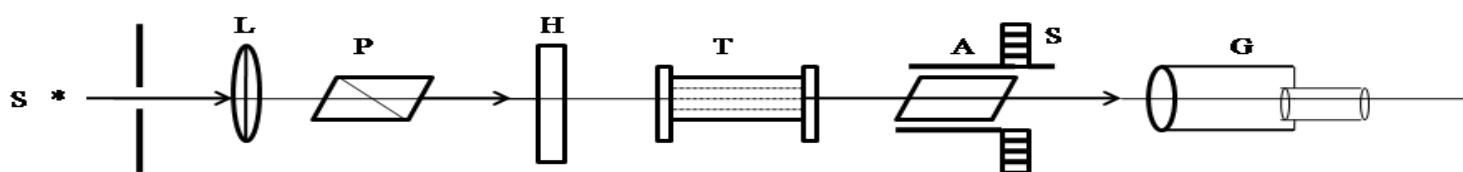


Fig.4.6

#### Working

Half -shade device consists of a semicircular half wave plate ACB of quartz cemented to a semicircular glass plate ADB. The optic axis of the wave plate is parallel to the line of separation AB. The thickness of the glass plate is such that it absorbs the same amount of light as the half wave plate. The light from the polariser (P) is plane polarized and falls normally on the half - shade plate. Let OP be the direction of vibrations in the plane

polarized which is inclined at an angle  $\theta$  to the optic axis AB. On passing through the glass half, the vibrations will remain along OP, but on passing through the quartz half, the vibrations will split into O-ray and E-ray. The vibrations of O-ray are along OD and E-ray vibrations along OA. The half wave plate introduces a phase difference of  $\pi$  radian between the two vibrations. Thus on emergence from the quartz plate, the vibration of O-ray will occur along OC instead of OD and the E-ray has vibrations still along OQ. Thus the light emerging from the quartz plate has resultant vibration along OQ such that  $\angle POA = \angle QOA = \theta$ . Thus the effect of quartz plate is to rotate the plane of polarization by an angle  $2\theta$ .

Now there are two plane polarized beams; one emerges from the glass plate with vibrations in the plane OP and the other emerges from the quartz plate with vibrations in the plane OQ. If the principal plane of the analyzing Nicol is parallel to QOQ', the light from the quartz plate will pass through the analyser. But the light from the glass plate will be partly stopped by the analyser. Hence quartz half will be brighter than the glass half. If the principal plane of the analyser is parallel to POP, the light from the glass plate will pass through the analyser. But the light from the quartz plate will be brighter than the quartz half.

When the principal plane of the analyser be parallel to AOB, it is equally inclined to the two plane polarized beams. Hence the two halves of the field appear equally illuminated.

## UNIT V

### KINETIC THEORY OF GASES

#### 5.1 CONCEPT OF HEAT AND TEMPERATURE

##### *System*

A definite quantity of matter bounded by some closed surface is known as system. Example: A gas contained in a cylinder provided with a movable piston, a hot solid ball in space.

##### *Surrounding*

Anything outside the system, which can exchange energy with it and has a direct influence on the behavior of the system, is called surroundings. The system and its surroundings together are called the universe.

##### *Internal energy*

According to kinetic theory of matter, matter is made up of molecules which are in a state of motion. The total K.E of the molecules forming a system is called the internal K.E of the system. Also the molecules possess P.E due to intermolecular force of attraction. The total P.E of molecules forming the system is known as its internal P.E. The sum of the internal K.E and internal P.E is known as internal energy ( $U$ ) of the system.

#### 5.2 Concept of Heat

Heat is defined as the energy in motion. Whenever a body is heated, the flow of energy to the body causes its internal energy to increase. The increase of internal energy produces a corresponding increase of temperature. Similarly, withdrawal of heat from a body causes lowering of internal energy and hence a corresponding lowering of its temperature. The flow of energy from one body to another body is possible only when there is a difference of temperature between them. Consider two

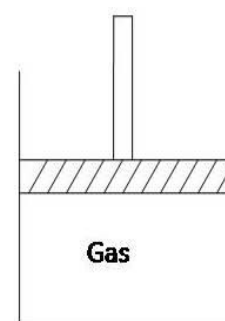


Fig 5.1

bodies at two different temperatures. When they are made to be in contact with each other, heat flows from the hot body to the body of lower temperature. Thus the hot body cools and

the cold body is heated. When the temperature of the two bodies becomes equal and there is no flow of heat from one body to the other, now the two bodies are said to be in thermal equilibrium with each other.

### **Specific heat capacity**

It is defined as the quantity of heat required to heat unit mass of substance through one Kelvin temperature is expressed by  $\text{Jkg}^{-1} \text{K}^{-1}$ .

The quantity of heat (Q) required to heat a substance of mass (m) through temperature  $t^\circ\text{C}$  is

$$Q = m.c.t$$

$Q = \text{mass} \times \text{specific heat capacity} \times \text{change of temperature}$ . The specific heat of a substance is different at different temperatures. If the temperature of a substance of mass m is raised through a small amount  $dT$  from  $T^\circ\text{C}$ , by applying an amount of heat  $dQ$  to it, then its specific heat capacity at temperature  $T^\circ\text{C}$  is given by

$$C = \frac{1}{m} \left( \frac{dQ}{dT} \right).$$

The specific heat capacity of the same substance is different at its different states. Specific heat capacity of ice =  $2100 \text{ Jkg}^{-1} \text{K}^{-1}$ ; specific heat capacity of water =  $4200 \text{ Jkg}^{-1} \text{K}^{-1}$

## **5.3 Concept of temperature**

A gas consists of a large number of similar molecules. They move in all directions with all possible velocities. During motion they collide with each other as well as with the walls of the container. The mean distance travelled by a molecule between two successive collisions is known as mean free path of the gas. Due to collision, the molecules exchange momentum among themselves and with the container. The rate of change of momentum of the molecules colliding per unit area of the walls of the container gives the pressure of the gas.

The molecules are assumed to be point masses. The volume of the gas is equal to the volume of the container. The molecules of a given mass of gas have K.E as well as P.E. The sum of K.E and P.E gives the internal energy of the gas.

For a perfect gas, the P.E is zero. Hence the internal energy of the gas is due to K.E of the molecules. The internal energy manifests as heat and thus heat is energy in motion. The degree of hotness is measured by the quantity known as temperature of the gas. At 0K, the internal energy of the gas is zero and the molecular motion stops. As the temperature increases, the internal energy also increases. This is the kinetic theory of concept of temperature.

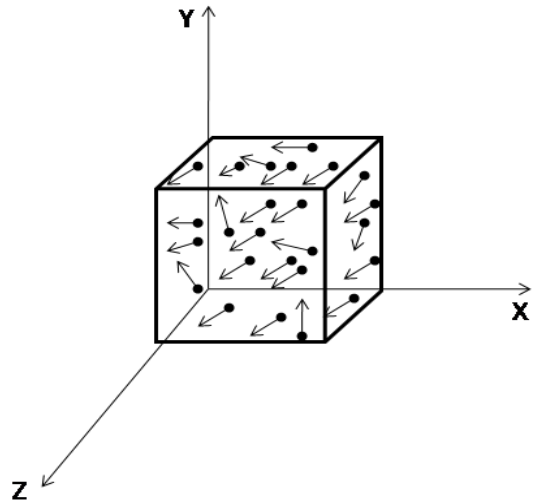


Fig. 5.2

### 5.4 Ideal and perfect gas

1. An ideal gas obeys Boyle's law and Charle's law. It obeys the gas equation  $PV = nRT$ . Where R is universal gas constant.
2. The internal energy U is independent of pressure and volume for a perfect gas. U is a function of temperature  $U = U(T)$ .

$$\left(\frac{\partial U}{\partial V}\right)_T = 0$$

3. There is no inter molecular force of attraction among molecules of a perfect gas. ie. the potential energy of molecules = 0. So the internal energy U is only due to kinetic energy of the molecules of the gas.

### 5.5 Postulates of kinetic theory of gases

1. A gas is composed of identical molecules. They behave as hard rigid and perfectly elastic spheres.
2. The volume of the molecules is negligibly small compared to the volume of the container.
3. The molecules move about in random directions. They have velocities ranging from zero to infinity.
4. During the random motion, the molecules collide with each other and also with the walls of the containing vessel.
5. The time of collision is negligibly small.
6. The average distance travelled by a molecule between two successive collisions is known as "Mean free path". Between collisions, a molecule moves in a straight line.

7. The molecules exert no force of attraction or repulsive force between them.
8. The collisions are assumed to be perfectly elastic. ie. There is no loss of K.E.

## 5.6 Expression for pressure exerted by a gas

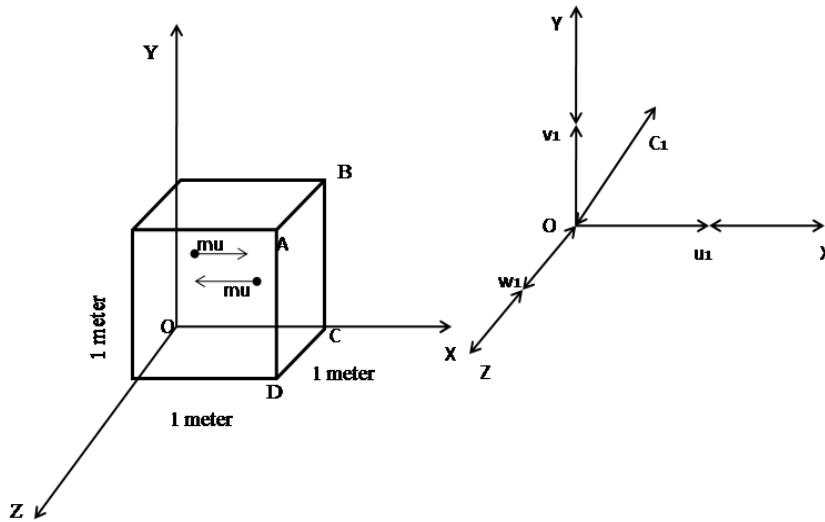


Fig.5.3(a)

Fig. 5.3(b)

Consider a gas contained in a cubic vessel of sides one meter which are parallel to X, Y and Z axes respectively. Let  $n$  be the number of molecules per unit volume and  $m$  be the mass of each molecule. The molecules make random motion and they collided with each other and with the walls of the container. Due to collision, the molecules exchange momentum among themselves and with the container. The rate of change of momentum is the force and force per unit area of the wall is the pressure exerted by the gas.

Consider the motion of one molecule with velocity  $C_1$ . The velocity can be resolved into three components  $u_1$ ,  $v_1$  and  $w_1$  among the three axes respectively.

$$C_1^2 = u_1^2 + v_1^2 + w_1^2$$

The molecule colliding with the wall ABCD of the cube with component velocity  $u_1$  will be rebounded with a velocity having the same magnitude but in opposite direction.

Momentum of the molecule before collision =  $mu_1$

Momentum of the molecule after collision =  $-mu_1$

Change of momentum per collision =  $mu_1 - (-mu_1) = 2mu_1$



The next collision with the same wall takes place after the molecule travels a distance of 2 meters along the X- axis.

$$\text{The time taken } t = \frac{\text{distance}}{\text{velocity}} = \frac{2}{u_1}$$

$$\text{Collision frequency} = 1/t = \frac{u_1}{2}$$

$$\text{The rate of change of momentum} = 2 mu_1 \times \frac{u_1}{2}$$

$$\therefore \text{the force imparted on the wall} = mu_1^2$$

Similarly,  $mu_2^2, mu_3^2, mu_4^2, \dots \dots \dots mu_n^2$  are the forces exerted on the wall ABCD due to collisions of molecules having velocities  $u_2, u_3 \dots \dots u_n$ .

The total force exerted on ABCD of unit area perpendicular to X- axis is the pressure  $P_x$ .

$$P_x = mu_1^2 + mu_2^2 + mu_3^2 + \dots mu_n^2$$

$$P_x = m(u_1^2 + u_2^2 + u_3^2 + \dots u_n^2)$$

Similarly, the pressure exerted on the wall, perpendicular to Y and Z axes are

$$P_y = m(v_1^2 + v_2^2 + v_3^2 + \dots v_n^2)$$

$$P_w = m(w_1^2 + w_2^2 + w_3^2 + \dots w_n^2)$$

The pressure exerted by a gas is same in all directions.

$$\text{ie., } P_x = P_y = P_w = P$$

$$P_x + P_y + P_w = 3P$$

$$3P = m(u_1^2 + u_2^2 + u_3^2 + \dots u_n^2) + m(v_1^2 + v_2^2 + v_3^2 + \dots v_n^2) + m(w_1^2 + w_2^2 + w_3^2 + \dots w_n^2)$$

$$= m[(u_1^2 + u_2^2 + u_3^2 + \dots u_n^2) + (v_1^2 + v_2^2 + v_3^2 + \dots v_n^2) + (w_1^2 + w_2^2 + w_3^2 + \dots w_n^2)]$$

$$= m[(u_1^2 + v_1^2 + w_1^2) + (u_2^2 + v_2^2 + w_2^2) + \dots + (u_n^2 + v_n^2 + w_n^2)]$$

$$= m (C_1^2 + C_2^2 + C_3^2 + \dots C_n^2)$$

$$= mn \frac{(C_1^2 + C_2^2 + C_3^2 + \dots C_n^2)}{n}$$

$$3P = mn\bar{c}^2$$

Where  $\bar{c}^2$  is the mean square velocity of the molecules in the gas.

$$P = \frac{1}{3} mn\bar{c}^2$$

If  $\rho$  be the density of the gas,  $\rho = \frac{\text{mass}}{\text{volume}} = \frac{mn}{1} = m n$ .

$$P = \frac{1}{3} \rho \bar{c}^2$$

This gives the pressure exerted by the gas molecules on the container.

## 5.7 Interpretation of temperature

### 1. To find the mean K.E of one molecule

$$P = \frac{1}{3} mn\bar{c}^2$$

$$mn\bar{c}^2 = 3P$$

$$m\bar{c}^2 = \frac{3P}{n}$$

from gas equation

$$\frac{1}{2} m\bar{c}^2 = \frac{3P}{2n}$$

PV=RT ; but R = n k

$$\therefore \text{Mean K.E per molecule} = \frac{3P}{2n}$$

$$PV = n k T$$

$$= \frac{3nkT}{2n}$$

$$P = \frac{nkT}{v} = n k T. \text{ Since volume is unity.}$$

$$= \frac{3kT}{2}$$

ie., mean K.E  $\propto$  T.

The mean K.E due to translational motion of the molecule is directly proportional to the absolute temperature of the gas. Thus K.E is zero at absolute zero temperature, the molecule ceases to have translatory motion at zero temperature. When heat content of the gas increases,

the average kinetic energy of the molecules in the gas increases. Thus, the average kinetic energy can be considered to be a measure of the temperature of the gas. This is the kinetic theory concept of temperature.

$$\text{K.E} \propto T \quad \therefore \frac{1}{2} m \overline{c^2} \propto T \text{ and } \overline{c^2} \propto T$$

ie., the mean square velocity of the molecules is directly proportional to the Kelvin temperature of the gas.

## 2. To calculate the root mean square (rms ) velocity of the gas

$$\text{r.m.s velocity} = \sqrt{\overline{c^2}}$$

$$P = \frac{1}{3} \rho \overline{c^2}$$

$$\overline{c^2} = \frac{3P}{\rho}$$

$$\sqrt{\overline{c^2}} = \sqrt{\frac{3P}{\rho}}$$

$$\text{Root mean square velocity} = \sqrt{\frac{3P}{\rho}}$$

We know,

$$PV = RT$$

When temperature is constant; PV= constant

$$\therefore \frac{P}{\rho} = \text{constant and}$$

$$\sqrt{\frac{3P}{\rho}} = \text{constant}$$

$\therefore$  rms velocity = constant

$$\text{also } C_{\text{rms}} = \sqrt{\frac{3P}{\rho}}$$

$$P = nkT$$

$$= \sqrt{\frac{3nkT}{\rho}}$$

$$\rho = mn$$

$$C_{rms} = \sqrt{\frac{3nkT}{mn}}$$

$$C_{rms} = \sqrt{\frac{3kT}{m}}$$

Thus,  $C_{rms} \propto T$

$$C_{rms} = \sqrt{\frac{3NkT}{mN}}$$

Since  $mN = \text{molecular weight} = M$  and  $Nk = R$ , the gas constant

$$C_{rms} = \sqrt{\frac{3RT}{M}}$$

### 3. Deduction of gas law

#### (a) Boyle's law

From kinetic theory of gas

The pressure of the gas  $P = \frac{1}{3} \rho \overline{c^2}$

$$P = \frac{1}{3} \frac{M}{V} \overline{c^2}$$

$$PV = \frac{1}{3} M \overline{c^2}$$

At constant temperature,  $\overline{c^2}$  is constant.

$$\therefore \frac{1}{3} M \overline{c^2} \text{ is constant}$$

$$PV = \text{constant}$$

i.e., pressure of the gas is inversely proportional to the volume. This is Boyle's law for gases.

#### (b) Charle's law

$$PV = \frac{1}{3} M \overline{c^2}$$

$$= \frac{1}{3} m N \overline{c^2}$$

Consider one mole of a gas at absolute temperature T,

$$PV = \frac{1}{3} m N \overline{c^2}$$

$$\text{The mean K.E} = \frac{1}{2} m \overline{c^2} = \frac{3kT}{2}$$

$$m \overline{c^2} = 3kT$$

$$PV = \frac{1}{3} N 3kT = NkT$$

$$PV = RT$$

Where N is the Avogadro number and K is the Boltzmann constant.

When P is kept constant,  $V \propto T$

When V is kept constant,  $P \propto T$ .

This is charle's law.

### **(c) Avogadro law**

Consider two gases A and B at pressure P and each having a volume V.

Mass of each molecules of gas A =  $m_1$

Number of molecules in gas A =  $N_1$

Mean square velocity of molecules in A =  $C_1^2$

$$\text{For this gas pressure } P_1 = \frac{1}{3} \rho_1 \overline{C_1^2}$$

$$= \frac{1}{3} \frac{m_1 n_1}{V} \overline{C_1^2}$$

Similarly, for the gas B,

$$P_2 = \frac{1}{3} \rho_2 \overline{C_2^2}$$

$$= \frac{1}{3} \frac{m_2 n_2}{V} \overline{C_2^2}$$

Now  $P_1 = P_2 = P$

$$\frac{1}{3} \frac{m_1 n_1}{v} \overline{C_1^2} = \frac{1}{3} \frac{m_2 n_2}{v} \overline{C_2^2}$$

$$m_1 n_1 \overline{C_1^2} = m_2 n_2 \overline{C_2^2} \dots\dots\dots(1)$$

If the two gases are at the same temperature T, the mean K.E of the molecules in both gases will be the same.

$$\frac{1}{2} m_1 \overline{C_1^2} = \frac{1}{2} m_2 \overline{C_2^2} \dots\dots\dots(2)$$

Dividing the eqn (1) by (2)

$$n_1 = n_2$$

Equal volume of all gases at the same temperature and pressure will contain the same number of molecules. This is Avogadro's law. The Avogadro number is N.

$$\begin{aligned} N &= 6.02 \times 10^{23} \text{ per mole} \\ &= 6.02 \times 10^{26} \text{ per kilo mole.} \end{aligned}$$

### 5. 8 Ideal gas equation

If M be the molecular weight, V be the molar volume then

$$P = \frac{1}{3} \rho \overline{C^2}$$

$$= \frac{1}{3} \frac{M}{V} \overline{C^2}$$

$$PV = \frac{1}{3} M \overline{C^2}$$

$$PV \propto \overline{C^2}$$

But  $\overline{C^2} \propto T$

$$PV \propto T$$

$$PV = RT$$

R is the gas constant.

In general, if V contains n moles

$$PV = nRT$$

## 5.9 Transport phenomena

The flow of material from one region to another is an example of a transport property. If a fluid is not in the equilibrium states, then any one of the following three phenomena may happen.

1. The velocities of different parts of the fluid may be different. So there may be a net transport of momentum from one part to another due to collisions among the molecules. This gives rise to the phenomenon of viscosity of the fluid.
2. The temperature of different layers of the fluid may be different. So there may be a net transport of thermal energy from one part to another. This gives rise to the phenomenon of thermal conductivity.
3. The concentration of different parts of the fluid may be different. So there may be a net transport of mass from one part to another. This gives rise to the phenomenon of diffusion in the fluid.

The above phenomena in which there is net transport of momentum, energy or mass from one part to another part of the fluid remaining in unsteady state, are known as transport phenomena.

### 5.9.1 Expression for viscosity (Transport of momentum)

Consider a gas flowing along the positive Y direction. The gas consists of different layers moving with different velocities. The velocity gradient along the positive Z direction be  $\frac{dv}{dz}$ .

Let v be the velocity of molecules in the layer A. Let B be another layer at a distance  $\lambda$  above A and C be the layer at a distance  $\lambda$  below A.  $\lambda$  is the mean free path of the molecules.

The velocity of molecules in layer A = V

The velocity of molecules in layer B =  $V + \lambda \frac{dv}{dz}$

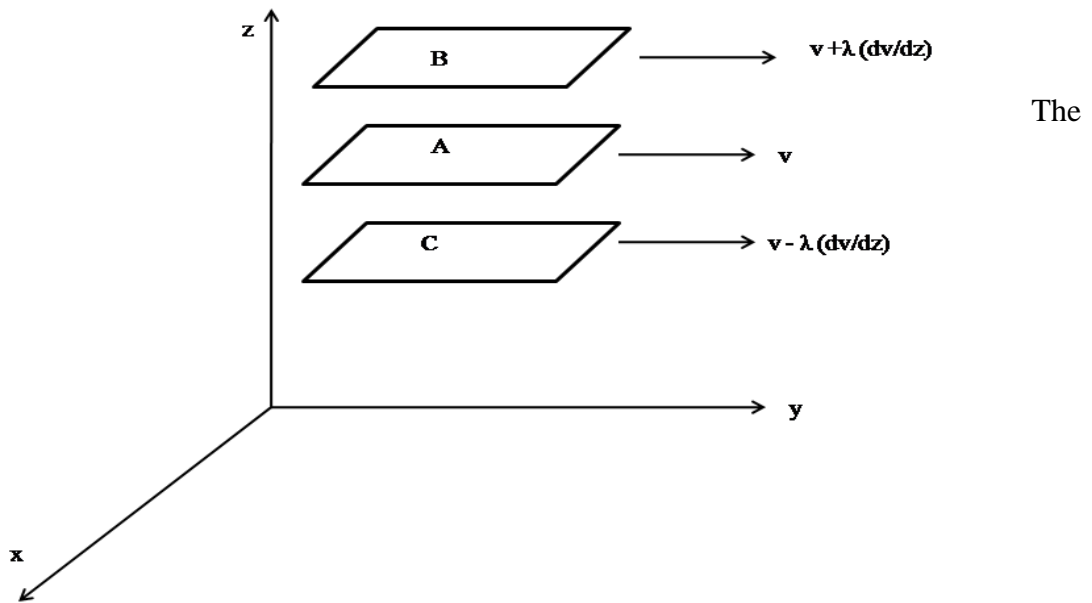


Fig.5.4

velocity of molecules in layer C =  $V - \lambda \frac{dv}{dz}$

Let  $n$  be the number of molecules per unit volume and  $m$  be the mass of each molecule. The possible number of molecules moving along  $Z$  direction is  $n/3$ . Half this number i.e.  $n/6$  may move upwards and half this number  $n/6$  move downwards. Let  $C$  be the mean velocity of the molecules of the gas.

The number of molecules crossing unit area of the layer A per second from above or below

$$= \frac{1}{6} nc$$

The mass of molecules crossing unit area of the layer A per second from above or below

$$= \frac{1}{6} ncm$$

The layer A gains due to collision of molecules from the layer B. there is no collision in between A and B.

The gain in momentum by layer A per unit area per unit sec

$$= \text{mass} \times \text{velocity}$$

$$= \frac{1}{6} ncm \left[ V + \lambda \frac{dv}{dz} \right]$$



The layer A loses momentum due to collision of molecules from the layer C.

The loss in momentum by layer A per unit area/ sec

$$= \frac{1}{6} ncm \left[ V - \lambda \frac{dv}{dz} \right]$$

The net gain in momentum by layer A per unit area/ sec

$$= \frac{1}{6} ncm \left[ V + \lambda \frac{dv}{dz} \right] - \frac{1}{6} ncm \left[ V - \lambda \frac{dv}{dz} \right]$$

$$= \frac{1}{6} ncm \left[ V + \lambda \frac{dv}{dz} - V + \lambda \frac{dv}{dz} \right]$$

$$= \frac{1}{6} ncm \times 2\lambda \frac{dv}{dz}$$

$$= \frac{1}{3} ncm \lambda \frac{dv}{dz}$$

This rate of change of momentum gives the tangential force per unit area between two layers having velocity gradient  $\frac{dv}{dz}$ . By definition  $\eta$  is the tangential force per unit area between two layers having unit velocity gradient. Hence  $\frac{dv}{dz} = 1$ .

We get the coefficient of viscosity of the gas as

$$\eta = \frac{1}{3} ncm \lambda$$

$$\eta = \frac{1}{3} \rho c \lambda \quad \text{since } \rho = nm$$

**(a) Dependence of  $\eta$  on the pressure of the gas**

When the pressure of a gas is increased at constant temperature, its density  $\rho$  also increases and correspondingly  $\lambda$  decreases. ie.  $\rho \propto \frac{1}{\lambda}$  and hence  $\rho \lambda$  remains constant when the pressure is increased. Thus  $\eta$  is independent of pressure.

**(b) Dependence of  $\eta$  on the temperature**

We note that  $\eta \propto C$

$$\text{But } C \propto \sqrt{T}$$

$$\text{and hence } \eta \propto \sqrt{T}$$

Thus  $\eta$  is dependence on square root of temperature.

**(c)  $\eta$  can be expressed in terms of molecular diameter  $d$**

$$\eta = \frac{1}{3} \rho c \lambda$$

$$\rho = nm \text{ and } \lambda = \frac{1}{\sqrt{2} \pi n d^2}$$

$$\text{now } \eta = \frac{1}{3} n m c \frac{1}{\sqrt{2} \pi n d^2}$$

$$\eta = \frac{m c}{3 \sqrt{2} \pi d^2}$$

By measuring the value of  $\eta$  of the gas at a particular temperature and knowing the mass of a molecule  $m$  and the mean velocity  $c$ , one can calculate the molecular diameter  $d$ .

### 5.9.2 Expression for thermal conductivity of a gas (transport of energy)

Consider a given mass of gas remaining at rest. The gas consists of parallel layers. Let the temperature of these layers be different. The temperature gradient along the  $Z$  axis be  $\frac{dT}{dz}$ .

Let  $T$  be the temperature of molecules in the layer  $A$ . Let  $B$  be another layer at a distance  $\lambda$  above  $A$  and  $C$  be the layer at a distance  $\lambda$  below  $A$ . Due to molecular motion, there will be transport of thermal energy from one layer to other.

Each molecule from  $B$  passing  $A$  downward have K.E higher than a molecule crossing  $A$  upwards. Thus the gas is conducting heat from  $B$  to  $C$ . Let  $n$  be the number of molecules per unit volume and  $m$  be the mass of each molecule. The possible number of molecules moving along  $Z$  direction is  $n/3$ . Half this number i.e.  $n/6$  may move upwards and half this number  $n/6$  move downwards. Let  $C$  be the mean velocity of the molecules of the gas.

The number of molecules crossing unit area of the layer  $A$  per second from above or below

$$= \frac{1}{6} n c$$

Let  $E$  be the mean energy of a molecule in the layer  $A$ . Let  $\frac{dE}{dz}$  be the energy gradient in the upward direction to the layer  $A$ .

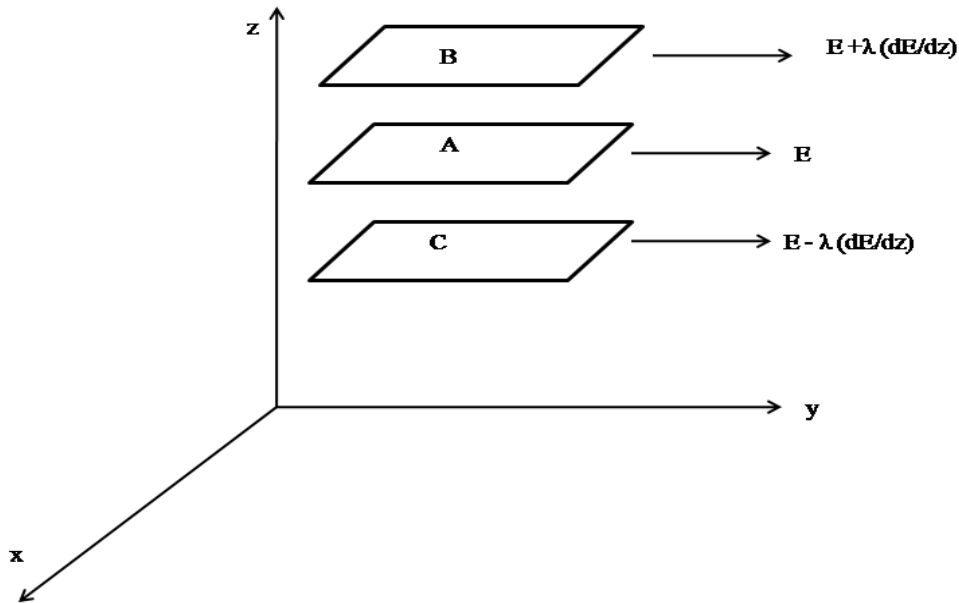


Fig.5.5

The mean energy per molecule at the layer B =  $E + \lambda \frac{dE}{dz}$

The mean energy per molecule at the layer C =  $E - \lambda \frac{dE}{dz}$

The energy carried downward by the molecules crossing unit area of layer A per second

$$\begin{aligned}
 &= \frac{1}{6}nc\left[E + \lambda\frac{dE}{dz}\right] - \frac{1}{6}nc\left[E - \lambda\frac{dE}{dz}\right] \\
 &= \frac{1}{6}nc\left[E + \lambda\frac{dE}{dz} - E + \lambda\frac{dE}{dz}\right] \\
 &= \frac{1}{6}nc \times 2\lambda\frac{dE}{dz} \\
 &= \frac{1}{3}nc\lambda\frac{dE}{dz}
 \end{aligned}$$

But  $dE = mc_v dT$

$c_v$  – specific heat at constant volume.

$$\frac{dE}{dz} = mc_v \frac{dT}{dz}$$

∴ The energy transported downwards per unit area per second

$$= \frac{1}{3}ncmc_v \lambda \frac{dT}{dz}$$

For unit temperature gradient ie.  $\frac{dT}{dz} = 1$

The energy transported across unit area per second

$$= \frac{1}{3} n c m c_v \lambda$$

By definition, this gives the thermal conductivity K

$$K = \frac{1}{3} n m c \lambda c_v$$

$$K = \frac{1}{3} \rho c \lambda c_v \quad \text{since } \rho = n m$$

Thus the thermal conductivity of a gas is proportional to the density of the gas, mean velocity of the molecules, the mean free path and the specific heat at constant volume of the gas.

**(a) Relation connecting K and  $\eta$**

We have  $K = \frac{1}{3} \rho c \lambda c_v$

And  $\eta = \frac{1}{3} \rho c \lambda$

Hence  $\frac{K}{\eta} = c_v$

**(b) Relation connecting K and molecular distance d**

Thermal conductivity  $K = \frac{1}{3} \rho c \lambda c_v$

Mean free path  $\lambda = \frac{1}{\sqrt{2} \pi n d^2}$

$$K = \frac{1}{3} \rho c \frac{1}{\sqrt{2} \pi n d^2} c_v$$

$$K = \frac{1}{3} n m c \frac{1}{\sqrt{2} \pi n d^2} c_v$$

$$K = \frac{m c c_v}{3 \sqrt{2} \pi d^2}$$

Here d is the molecular diameter. This shows K is independent of pressure and varies directly as the square root of the Kelvin temperature. ie.  $K \propto \sqrt{T}$ .

Since  $C = \left(\frac{8KT}{\pi m}\right)^{\frac{1}{2}}$

K is inversely proportional to the square root of mass of the molecule.

### 5.9.3 Expression for coefficient of diffusion

Consider a gas in which the molecular concentration varies in the vertical direction but

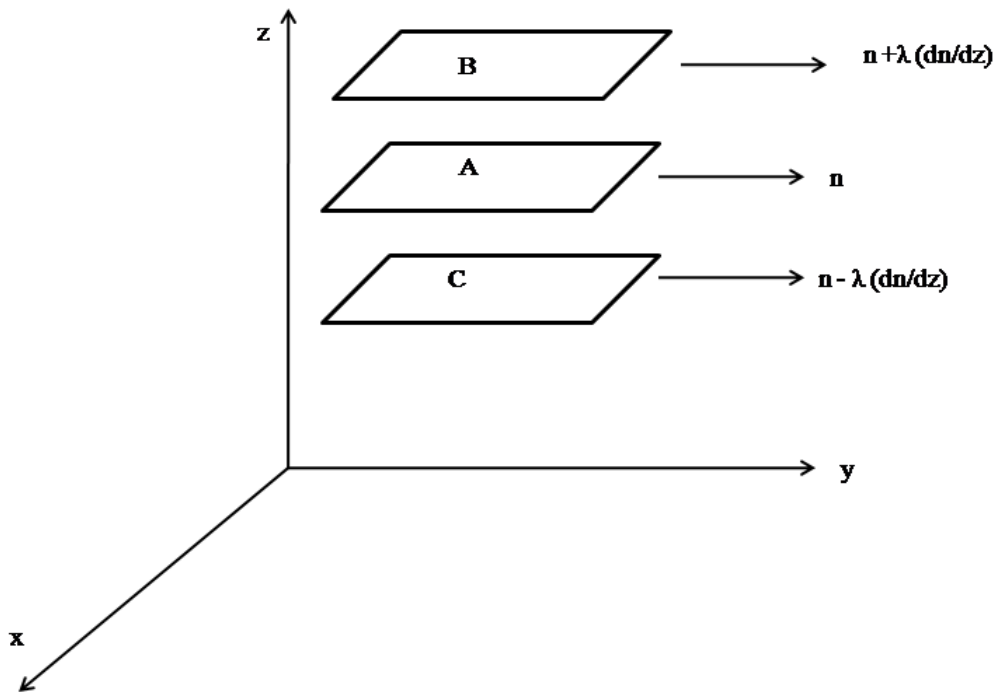


Fig.5.6

remains constant in a horizontal plane. Let the concentration gradient along Z axis be  $\frac{dn}{dz}$ .

Let n be the concentration of the molecules in layer A. Let B be another layer at a distance λ above and C be another layer at a distance λ below A.

Concentration in layer A = n

Concentration in layer B =  $n + \lambda \frac{dn}{dz}$

Concentration in layer C =  $n - \lambda \frac{dn}{dz}$

Let m be the mass of each molecule and C be the mean velocity.

The number of molecules crossing unit area of the layer A per unit second from above

$$= \frac{1}{6} c \left[ n + \lambda \frac{dn}{dz} \right]$$

The number of molecules crossing unit area of the layer A per unit second from below

$$= \frac{1}{6} c \left[ n - \lambda \frac{dn}{dz} \right]$$

The number of molecules moving downwards across unit area per unit second from

$$= \frac{1}{6} c \left[ n + \lambda \frac{dn}{dz} \right] - \frac{1}{6} c \left[ n - \lambda \frac{dn}{dz} \right]$$

$$= \frac{1}{6} c \left[ n + \lambda \frac{dn}{dz} - n + \lambda \frac{dn}{dz} \right]$$

$$= \frac{1}{6} c \left[ 2 \lambda \frac{dn}{dz} \right]$$

$$= \frac{1}{3} c \lambda \frac{dn}{dz}$$

By definition, the coefficient of diffusion is the number of molecules crossing unit area of a plane per unit time under unit concentration gradient perpendicular to the plane. Hence by putting  $\frac{dn}{dz} = 1$

$$D = \frac{1}{3} c \lambda.$$

Here,  $C$  is the mean velocity and  $\lambda$  is the mean free path of the molecules of the gas.

$C$  is inversely proportional to the square root of mass of molecules and hence to the square root of molecular weight. Hence  $D$  is inversely proportional to the square root of molecular weight of the gas. Hence hydrogen whose molecular weight is sixteen times smaller than that of oxygen diffuses four times as fast as oxygen.

### ***Relation between $D$ and $\eta$***

We have,

$$D = \frac{1}{3} c \lambda \quad \text{and}$$

$$\eta = \frac{1}{3} \rho c \lambda ;$$

hence,

$$D = \frac{\eta}{\rho}$$

We know  $\lambda \propto \frac{1}{n}$  and  $\lambda \propto \frac{T}{P}$ ; also  $C \propto \sqrt{T}$

Using these proportionalities, we get

$$D = \frac{1}{3} c \lambda \quad \text{becomes}$$

$$D \propto \sqrt{T} \times \frac{T}{P}$$

$$D \propto \frac{T^{\frac{3}{2}}}{P}$$

ie. D is directly proportional to  $T^{\frac{3}{2}}$  and inversely proportional to the pressure P of the gas.

## 5.10 Behavior of gases at high pressures

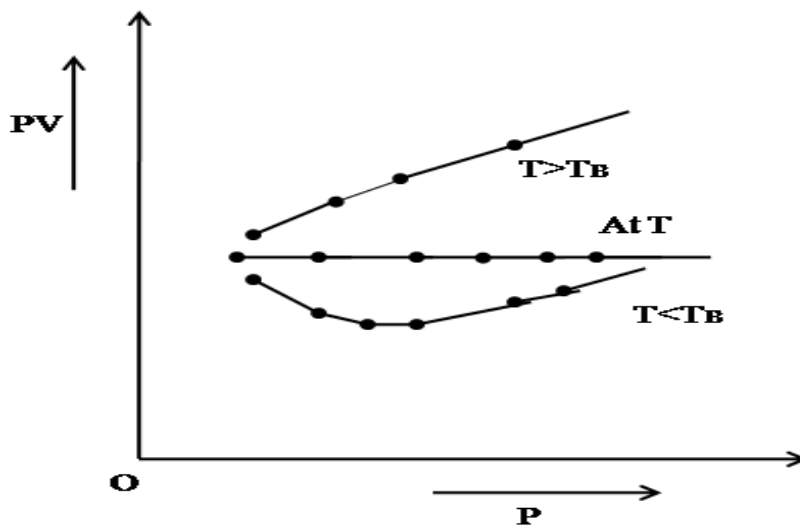


Fig 5.7

1. Actual gases do not obey Boyle's law at high pressures.
2. The deviation from Boyle's law follows the same pattern for all gases.
3. When a gas is brought to or below its critical temperature  $T_C$ , it gets liquefied on application of suitable high pressure. At low temperatures ( $T < T_c$ ), there is sudden decrease of PV with increase of pressure P. This corresponds to the change of state from gas to liquid.

#### 4. Boyle's temperature

Consider a gas at constant temperature is compressed at high pressure. The graph plotted between PV value and pressure P shows that, PV varies with P, depending on its temperature. At one particular temperature  $T_B$ , the value of the product PV is constant whatever be the pressure. This temperature at which a gas obeys Boyle's law is known as Boyle's temperature.

5. At high temperatures, the value of PV increases with increase of pressure.
6. At lower temperatures, the value of PV first decreases when pressure is increased. It reaches a minimum at particular pressure and then increases with increase of pressure. Below Boyle's temperature, the gases are highly compressible.

$$\text{Compressibility } K = \frac{dV}{V.dP}$$

This suggests the existence of intermolecular attraction. Above Boyle's temperature the intermolecular attractions are less significant.

### 5.11 Critical constants

#### ***Critical temperature***

The critical temperature  $T_c$  of a substance is that temperature above which a gas cannot be liquefied, even high pressure is applied.

#### ***Critical volume***

The critical volume  $V_c$  is the volume occupied by unit mass of the substance at the critical temperature and pressure.

#### ***Critical pressure***

Critical pressure  $P_c$  is the pressure at the critical point and it is the minimum pressure that must be applied in order to liquefy the gas at its critical temperature.

The quantities  $T_c$ ,  $V_c$  and  $P_c$  are known as critical constants of the substance under consideration.



## 5.12 Experimental determination of critical constants

The apparatus consists of a long tube bent into the form of letter J and fixed short and wide tube B. the ends of tubes are sealed. The tube A contains mercury and the tube B contains the experimental liquid and its saturated vapor. The air and liquid are separated by mercury column. The A containing air serves as compressed air manometer to measure pressure of saturated vapor in B. the apparatus is placed in an oil bath and gradually heated. As the temperature is gradually increased, the meniscus of the liquid becomes flatter and flatter and at certain temperature the free surface just disappears. As the bulb is allowed to cool, the meniscus suddenly re-appears at a particular temperature. The mean of the two temperatures is the critical temperature  $T_c$  of the gas. The pressure as read in the manometer gives the critical pressure  $P_c$ .

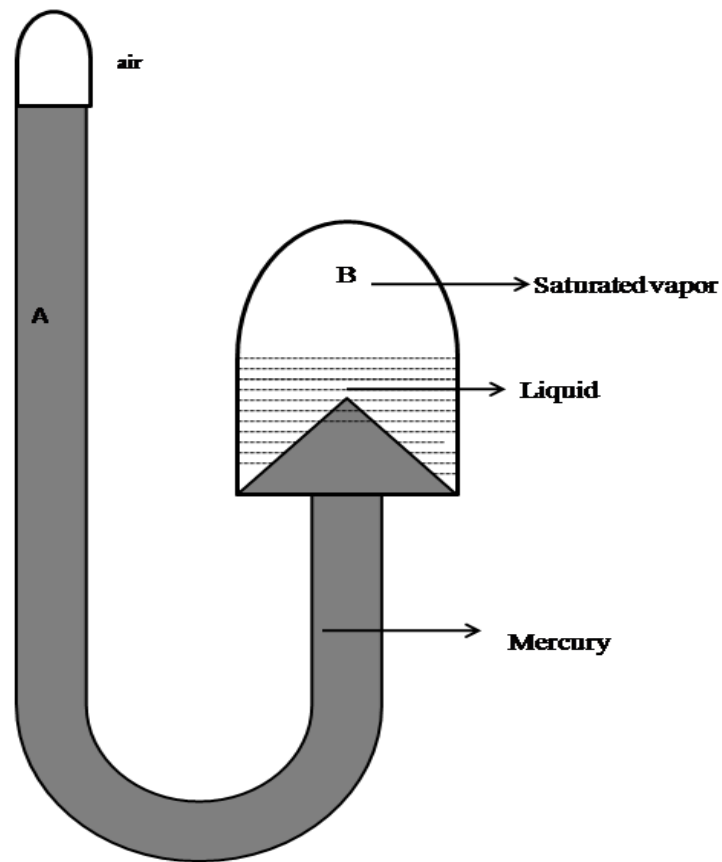


Fig.5.8

The critical volume can be found by applying the principle of rectilinear diameters due to Cailletet and Mathias. The densities of the liquid and of its saturated vapor are determined at different temperatures and are represented graphically as shown in figure 5.9.

The density of liquid decreases with increase of temperature but the density of saturated vapor increases with temperature. At different temperatures, the mean density is marked on the

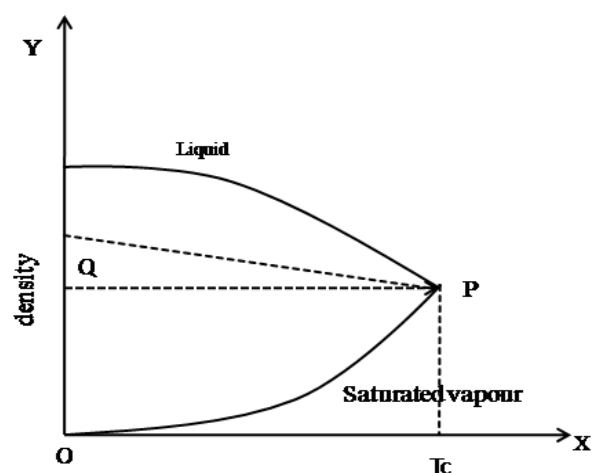


Fig.5.9

graph. The points lie on a straight line and it is known as rectilinear diameter. The line is produced and the curves are extra plotted to meet the line at P. The density corresponding to the point P is obtained from the graph. If the density is  $\rho_c$ , the specific volume of the gas. The temperature corresponding to the point p gives the critical temperature  $T_c$  of the gas.

### 5.13 Expression for critical constants in terms of Vander Waal's constants

Consider the isothermal graph of the critical temperature  $T_c$ . The point C on the graph is a point of inflection, where the rate of change of pressure with volume is zero.  $\frac{dP}{dV} = 0$ . The values of volume, pressure and temperature corresponding to the point C are called the critical constants. There are the critical volume ( $V_c$ ), critical pressure ( $P_c$ ) and critical temperature ( $T_c$ ).

At  $T = T_c$ ,  $\frac{dP}{dV} = 0$  and  $\frac{d^2P}{dV^2} = 0$  .....(1)

By Vander Waal's equation for actual gases,

$$(P + \frac{a}{V^2})(V - b) = RT$$

$\therefore P = \frac{RT}{(V-b)} - \frac{a}{V^2}$  .....(2)

$$\frac{dP}{dV} = \frac{RT}{(V-b)^2} + \frac{2a}{V^3}$$

$$\frac{d^2P}{dV^2} = \frac{2RT}{(V-b)^3} - \frac{6a}{V^4}$$
 .....(3)

At the critical point,  $\frac{dP}{dV} = 0$ ;  $V = V_c$ ;  $P = P_c$  and  $T = T_c$

$$\frac{-RT_c}{(V_c-b)^2} + \frac{2a}{V_c^3} = 0$$

$$\frac{RT_c}{(V_c-b)^2} = \frac{2a}{V_c^3}$$
 .....(4)

Also,  $\frac{d^2P}{dV^2} = 0$  at C.  $\therefore$  from equation (3)

$$\frac{2RT_c}{(V_c-b)^3} - \frac{6a}{V_c^4} = 0$$

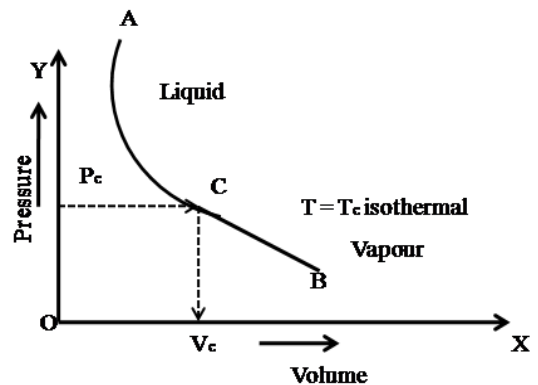


Fig.5.10

$$\frac{2RT_c}{(V_c-b)^3} = \frac{6a}{V_c^4} \dots \dots \dots (5)$$

Dividing equation (4) by (5),

$$\frac{V_c-b}{2} = \frac{V_c}{3}$$

$$3V_c - 3b = 2V_c$$

$$V_c = 3b$$

Substituting this eqn (4),

$$\frac{RT_c}{(2b)^2} = \frac{2a}{(3b)^3}$$

$$\therefore T_c = \frac{8a}{27Rb}$$

Putting this value and  $V_c = 3b$  in equation (2)

$$P_c = \frac{RT_c}{3b-b} - \frac{a}{(3b)^2}$$

$$P_c = \frac{8aR}{27Rb \times 2b} - \frac{a}{9b^2}$$

$$P_c = \frac{a}{27b^2}$$

Thus , the critical constants are

$$V_c = 3b ; T_c = \frac{8a}{27Rb} ; P_c = \frac{a}{27b^2} .$$

**Calculation of Vander Waal's constants a, b in terms of critical constants**

$$\frac{T_c^2}{P_c} = \left(\frac{8a}{27Rb}\right)^2 \times \frac{27b^2}{a} = \frac{64a}{27R^2}$$

$$\therefore a = \frac{27R^2 T_c^2}{64P_c}$$

Next ,

$$\frac{T_c}{P_c} = \frac{8a}{27Rb} \times \frac{27b^2}{a} = \frac{8b}{R}$$

$$\therefore b = \frac{RT_c}{8P_c}$$

## 5.14 The Joule –Thomson's porous plug experiment

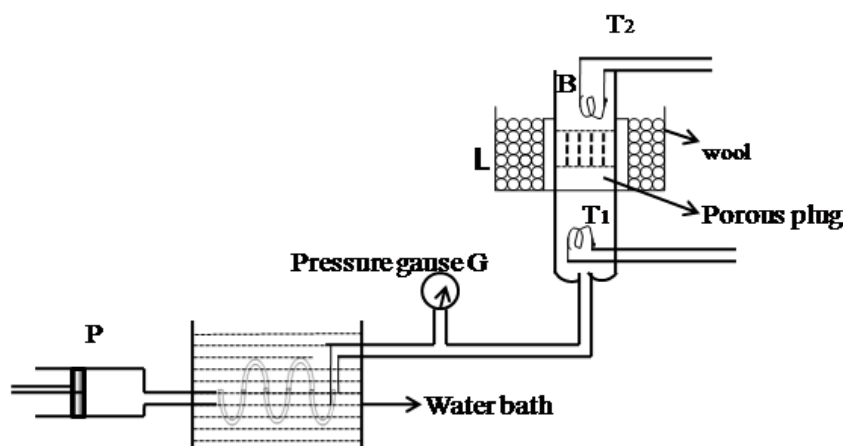


Fig.5.11

The porous plug material like silk cotton –wool, kept in position by two perforated brass disc. The porous plug has number of fine holes and it is kept in a cylindrical wooden tube. The wooden tube is surrounded by a jacket L which is packed with cotton wool. This is to avoid heat exchange with the surroundings.

The experimental gas is compressed to a required high pressure using the piston P. The heat of compression is removed by passing the gas through a spiral tube S immersed in a constant temperature water bath. The compressed gas is allowed to pass through the porous plug. The gas gets slow down while passing through the cotton wool. The gas expands on the other side of the plug. The temperature of the incoming and outgoing gases is measured by platinum resistance thermometers  $T_1$  and  $T_2$ . The pressure of the incoming gas can be measured with a gauge and that of the outgoing gas is at atmospheric pressure. The experiment is done with different types of gases. The following results are obtained.

1. All gases showed a change in temperature on passing through the porous plugs.
2. At ordinary temperatures, all gases except hydrogen and helium showed cooling effect and showed heating effect at room temperature.
3. The fall in temperature is directly proportional to the difference of pressure on the two sides of the porous plug.
4. The fall in temperatures is decreased as the initial temperature of the gas increases.

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